Benchmark Index of Risk Appetite

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Abstract

Changes in investors’ risk attitudes have been used to explain diverse phenomena in asset markets. And yet, popular indicators of changes in risk appetite typically have scant foundation in theory, and give contradictory signals in practice. The question is which one of them, if any, captures changes in investors’ risk attitudes. The author, building on the work of Kumar and Persaud (2003) and Misina (2003), proposes a method of computing the index of risk appetite that satisfies theoretical conditions which ensure that it indeed captures these changes. This index is then used to evaluate commonly held views regarding the behaviour of risk attitudes during various financial episodes over the last 20 years, investigate the behaviour of ‘safe haven assets’, and assess some risk appetite indices used in practice.

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1 I would like to thank Mark Illing for valuable contributions in Section 4.
1. Introduction

Investors’ newsletters and daily reports are replete with stories of changing investors’ “risk appetite” and suggestions as to the best way to benefit from these changes. Part of the difficulty with these stories is that it is often unclear what exactly is meant by “risk appetite.” Broadly speaking, “risk appetite” seems to be a stand-in for market sentiment, but at this level of generality the concept is hard to operationalize. More precise meaning can be attached to the concept, but there are various possibilities:

– risk appetite refers to investors’ risk aversion
– risk appetite simply means demand for risky assets
– risk appetite refers to the quantity of risky assets demanded.

The second and third interpretations, while plausible, lead to non-informative statements about market developments, implying that asset prices have changed because demand for (quantity demanded of) risky assets has changed. Causes of changes in demand (quantity demanded) are not specified. From the point of view of mapping this concept into an asset pricing model, the first interpretation seems to be the easiest. However, this interpretation implies that, if practitioners’ explanations are to be taken seriously, agent's utility function is not constant, given that their attitudes towards risk are allowed to change.\footnote{Endogenously changing risk attitudes \textit{can} be accommodated within the standard framework. Habit persistence utility functions deliver risk attitude that depends on surplus consumption and changes over time as surplus consumption changes. This mechanism, however, is typically found unsatisfactory, given that practitioners use changing risk attitudes to explain \textit{sudden} movements in asset prices, or a shorter-term phenomena.} Since constant preferences are thought of as safeguarding rigour in academic research, the allusions to non-constant preferences are typically frowned upon in academic circles.
Gai and Vause (2004) and Misina (2005) tackle this difficulty by distinguishing between risk aversion, which is assumed to be constant, and risk appetite which is allowed to vary over time. Gai and Vause (2004) postulate that “risk aversion is part of the intrinsic make-up of the investor and is a parameter that does not change markedly, or frequently, over time.” Risk appetite, on the other hand, is “somewhat more than the notion of risk aversion”, and “shifts periodically as investors respond to episodes of financial distress and macroeconomic uncertainty.” Misina (2005) differentiates between investors’ risk attitude as specified in theoretical models by the Arrow-Pratt coefficient of risk aversion, and the risk attitude implied by agents’ actions. To describe the latter, the notion of implied risk aversion is introduced in the standard expected utility framework. Implied risk aversion can change over time. Moreover, the paper characterizes the change as a function of agents’ future outlook. In this way, the requirement of constant risk attitudes is reconciled with observed behaviour that seem to indicate otherwise.³

This may clarify conceptual issues, but there are practical problems. Identification of changes in risk appetite usually relies on some type of in-house index that is purported to capture investors’ changing attitude towards risk. Practitioners use a wide variety of risk appetite indices, yet, as the recent survey by Illing and Aaron (2005) shows, these indices give contradictory signals even though they are presumably capturing the same phenomenon. Depending on which indicator is used, it is possible to conclude that the same price change was due to either increasing or decreasing appetite for risk! These findings raise the question of which one of them, if any, in fact captures changing risk

appetite. More generally, is it possible to disentangle the effect of changes in risk and risk attitudes?

Part of the answer to this question, at a theoretical level, was provided by Misina (2003). Starting from a broad class of asset pricing models, the paper identifies the key condition needed to ensure that a particular index of changes in investors’ risk appetite, introduced by Kumar and Persaud (2003), will distinguish between changes in risk appetite and asset riskiness. The key condition needed to break the ‘observational equivalence’ is that cross-correlations of asset returns be zero, which implies a diagonal variance/covariance matrix of asset returns. The condition is arguably unlikely to be satisfied in practice, especially if attention is limited to financial assets. Moreover, even if one succeeds in finding two assets whose returns are uncorrelated, it would seem that the chances of finding an uncorrelated portfolio decrease significantly with each new addition of the asset.

The present work builds directly on that work and constructs a benchmark index of risk appetite that satisfies the key condition identified in that paper. The approach we take is based on the observation that although the requirement of zero-covariances among returns may be a strong one when the original returns data are used, one can reverse the procedure and transform the original data in such a way that the requirement of zero correlation is achieved. In other words, we propose to generate a new data set from the original data, in such a way that the desired property is satisfied. The rank correlation measure of risk appetite, based on the same method as in Kumar and Persaud

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4 Broadening our horizon to real assets may improve the chances of success somewhat but it is questionable how returns on these are to be measured.

One might be tempted to argue that looking at both bonds and equity increases our chances of finding a portfolio with diagonal variance-covariance matrix but this need not be the case. Whereas returns on stocks and bonds should not be positively correlated, the same argument does not imply a zero correlation but rather a negative correlation.
(2003), is then computed on the transformed data set rather than on the original one. Since the key condition is satisfied in transformed data, changes in the RAI would now indicate changing risk appetite. Our index can thus be used as a benchmark against which other risk appetite indices are to be assessed. As part of the assessment, we compare our index to the original Global Risk Appetite Index (GRAI) of Kumar and Persaud (2003) as well as other risk appetite indices. We also provide the evidence on the behaviour of risk appetite in major financial episodes over the last 20 years.

The paper is organized as follows. In Section 2 we briefly review the results given in Misina (2003) that motivate the rest of the paper. In Section 3 we discuss the data transformation proposed, the interpretation of the transformed data, and the relationship of this method to the APT model. In Section 4 we present the results, and compare our index with other indices in current use. Performance of the index in major financial episodes is investigated as well. The final section of the paper concludes.

2. Necessity of Independent Returns

Suppose an analyst observes a change in prices of assets in a portfolio, and tries to infer whether it was due to a change in riskiness of some assets, or to a change in investors’ risk appetite. For this task to be feasible, one must assume that these two different causes of asset price changes will not be observationally equivalent. In other words, it must be assumed that these two causes will result in different behaviour of asset prices. Kumar and Persaud (2003) introduce the following distinction: changes in investors risk appetite should impact all assets in the portfolio in proportion to their degree of riskiness. On the other hand changes in riskiness of any particular asset would not have systemic effects on returns of other assets in the portfolio. Kumar and Persaud then propose rank
correlation of excess returns and asset riskiness as a measure that would capture these effects. In particular, positive rank correlation would indicate that a change in prices is due to changes in risk appetite, while a zero correlation would indicate a change in prices due to changes in riskiness of a particular asset.

The soundness of the proposed measure hinges on the validity of the distinction. Is the proposed distinction valid? Misina (2003) identifies the conditions under which the answer to the question is positive. In that paper, the intuition offered by Kumar and Persaud is summarized in the following propositions:\(^5\)

**Proposition 2.1** A change in investors’ risk appetite will have monotonic effects on assets in different risk classes: the impact on returns will depend on the riskiness of a particular asset.

**Proposition 2.2** A change in the riskiness of an asset will not have monotonic effects on excess returns across different asset classes. The impact on returns will not depend on the riskiness of a particular asset.

Letting \( R_{ex}^k \) denote the excess return on a risky asset, \( \rho \) the coefficient of investors’ risk aversion, and \( \mu_k \) a measure of the riskiness of an asset in class \( k \), Proposition 1 states that, when there is a change in risk aversion, there will be a rank effect,

\[
\mu_j > \mu_l \Rightarrow \Delta R_{ex}^j > \Delta R_{ex}^l, \quad \forall j > l,
\]

when the risk aversion increases, and the opposite effect when it decreases. Quantitatively, this effect can be captured by the rank correlation. Proposition 2 states that this effect will not emerge when riskiness of assets changes.

The question is whether these propositions can be derived within a well-specified asset pricing model. The answer is positive. Using a simple consumption-based asset-

pricing model, the following expression is obtained:

\[
\begin{bmatrix}
R_{1}^{ex} \\
R_{K}^{ex}
\end{bmatrix} = \rho \begin{bmatrix}
\sigma_{1,W} \\
\vdots \\
\sigma_{K,W}
\end{bmatrix} \equiv \rho \begin{bmatrix}
\sigma_{1}^2 \\
\sigma_{1,K} \\
\sigma_{K,1} \\
\sigma_{K}^2
\end{bmatrix} \begin{bmatrix}
\alpha_{1} \\
\vdots \\
\alpha_{K}
\end{bmatrix}.
\]

(2)

In this setting, Proposition 1 can be proved without imposing any further restrictions. The effect of changes in risk aversion is given by

\[
\frac{\partial R_{i}^{ex}}{\partial \rho} = \sigma_{i,W}, \forall k.
\]

(3)

If portfolio assets are ordered in such a way that \(\sigma_{j,W} > \sigma_{l,W}, \forall j > l\), it follows that

\[
\Delta R_{j}^{ex} > \Delta R_{l}^{ex}, \forall j > l.
\]

This establishes Proposition 1.

To prove Proposition 2, further restrictions are needed. The key condition for rank correlation to be an indicator of changes in investors’ risk appetite is that asset returns be independent, and this condition is summarized by the diagonal variance-covariance matrix. Moreover, even when with diagonal variance-covariance matrix, the presence of common shocks such that \(d\sigma_{k}^2 > 0\), or \(d\sigma_{k}^2 < 0, \forall k\), may lead to emergence of rank effect even when risk aversion is held constant.

To implement this measure, one has to empirically satisfy the condition of independence of returns, and find a way to assess whether assets in the portfolio have been subject to common shocks at any given time. The first issue is dealt with in the following section. The second issue is addressed in Section 4.

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6 See Misina (2003, 9) or Cochrane (2001, 154) for details.

7 See Misina (2003, 10-16) for details and derivations.
3. Orthogonalization of Returns

It is clear that the requirement of independent returns is a strong one and unlikely to satisfied empirically. As any practitioner can attest, one can perhaps find a couple of assets whose returns are uncorrelated. Finding a whole portfolio of uncorrelated assets is extremely unlikely.

We propose to circumvent this problem by orthogonalizing the set of returns on the assets comprising a given portfolio. Suppose that the portfolio under consideration consists of \( K \) assets, and let \( R_k \) denote a \( T \times 1 \) vector of returns on asset \( k \). The return matrix for the portfolio is,

\[
R = \begin{bmatrix} R_1 & \ldots & R_K \end{bmatrix}.
\]

The transformation proposed here is based on the fact that if the space of returns is \( K \)-dimensional, there will be \( K \) orthogonal linearly independent vectors spanning it. Denote these vectors by \( F_k \). The basis vectors will be linearly independent and as such satisfy the zero cross-correlation condition. Moreover, each \( F_k \) can be written as a linear combination of asset returns:

\[
F_i = \sum_{k=1}^{K} \gamma_k^i R_k,
\]

for some values of \( \gamma_k^i \). One can interpret each of the basis vectors as a derivative asset formed from the original assets. The returns on these derivative assets are a linear combination of returns on given assets.\(^8\) For example, a derivative asset with return profile

\(^8\) One could interpret these vectors as Arrow securities as well, but one need not. Arrow securities do form the ‘usual’ basis of the returns space. We need not use this usual basis, but the orthogonality property of vectors in our new basis is preserved.
$F_1 = -R_1 + R_2$ would be obtained by going short on asset with return $R_1$ and long on asset with return $R_2$.

The method proposed here bears close resemblance to the APT theory of Cox, Ingersoll and Ross. The idea behind that theory is to postulate a linear mechanism that generates returns on all assets in the portfolio as a linear combination of a set of underlying factors. If we let $R_i$ denote returns on asset $i, i = 1, ..., I$, and $F_k$ the factors generating returns then the generating mechanism takes the form

$$R_i = \sum_{k=1}^{\kappa} \alpha_k^i F_k + \varepsilon_i, \quad \kappa < K, \forall i.$$ 

It is assumed that $\varepsilon_i \perp F_k, \forall k$. The error term is interpreted as idiosyncratic component of returns, which represents the non-systematic risk. Factors are assumed to capture the systematic risk. In this formulation, returns are almost spanned by the factors, but not quite due to the presence of the error term. We propose to extend the dimension of the factor space so that returns are exactly spanned by factors i.e.

$$R_i = \sum_{k=1}^{K} \alpha_k^i F_k, \quad \forall i.$$ 

The procedure for finding these factors is that of the factor analysis except that in the present case the attention is not limited to the first $k$ factors that are deemed the most important, but rather to all factors. The methodology used is the same as that of the APT but since our objectives are different we do not focus on factor loadings. Technical details and the transformation procedure are described in Appendix 1.

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9 See, for example, Ross (1976).
4. **Benchmark Index of Risk Appetite**

Using the above results, the rank correlation is computed on transformed data. Since the key condition is satisfied, we know that the results here will indicate changes in risk appetite, if any.

Figure 1: RAI-MI unsmoothed

Figure 1 represents the rank correlation between risk and excess returns for a portfolio of currencies, for the period 1981 - 2005.\(^{10}\) We label this index as RAI-MI to distinguish this index from the index computed by Kumar and Persaud (2003). The index is quite volatile, and, is typically reported after some smoothing has been applied to it. It has to be kept in mind, however, that the basis for the interpretation of the index is the claim that the values of rank correlation not significantly different from zero would indicate that the observed change in prices is due to changes in riskiness rather than risk attitudes. Values of the index statistically different from zero would indicate that a change in price is due to either decreasing risk aversion (positive correlation), or increasing risk aversion (negative correlation).

The hypothesis \(H_0 : \rho_S = 0\) is tested using the test statistic

\[
t = \frac{\rho_S \sqrt{K - 2}}{\sqrt{1 - \rho_S^2}},
\]

which, under the null, follows \(t\) distribution with \(K - 2\) degrees of freedom.\(^{11}\) The shaded area in Figure 1, where \(\rho_S \in [-0.12, 0.12]\), represents the values of rank corre-

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\(^{10}\) A description of the data and estimation procedure is contained in the technical appendix.

\(^{11}\) This is true under the assumption of bivariate normality. In a Monte Carlo study, Zimmerman, Zumbo, and Williams (2003) show that this test statistic is robust to departures from normality in tests \(H_0 : \rho_S = 0\).
lation for which \( H_0 \) cannot be rejected at 95 percent confidence level. Based on this, a broad characterization of the behaviour of investors’ risk attitudes is possible. 1980s were characterized by a relatively high appetite for risk in the early 80s, and pronounced volatility in the second half of the period. From 1989 to 1996 investors, with a couple of exceptions, displayed low appetite for risk, followed by an increased risk appetite from 1997 to late 2000. The last four years have been characterized by pronounced volatility not dissimilar to what was observed in late 1980s, with a bias towards low appetite for risk. These trends are more easily observed in the smoothed version of the index. Keeping in mind that there is a range of values of the index that are not statistically different from zero, in the ensuing discussion we will use the smoothed version of the index. Figure 2 gives the smoothed version of the index and also marks major financial events during the sample period.

Figure 2: RAI-MI smoothed

Figure 2 illustrates the trend of the MI from 1983 to 2005. Significant financial market events are annotated on the figure. These include: the adoption of the Plaza Accord in 1985; the stock market crash of October 1987; the collapse of the U.S. junk bond market (1990-1992) and the related bankruptcy of Drexel Burnham Lambert; Great Britain’s withdrawal from the European Exchange Rate Mechanism (ERM) in September 1992; the Peso Crisis beginning with Mexico’s debt default in late December 1994; the collapse of Long-term Capital Management (LTCM) in September 1998 following Russia’s debt default in the prior month (and market turmoil more broadly in the aftermath of the south east Asian crisis over the preceding year); the terrorist attacks on 11 September 2001; and the eruption of corporate scandals in 2001 and 2002 (involv-
ing, among others, Enron, Global Crossing, and WorldCom). Two episodes of financial market euphoria are also labelled on the figure: the peak of the bond market in 1997 and the peak of the 1990s stock market bubble.

The index indicates that the risk appetite was generally high around the peak of the corporate bond markets and the stock market bubble in the late 1990s. On the other hand, risk appetite was in the neutral territory around the time of the stock market crash in 1987. Note that these observations are not meant to validate the index. Its validity is established by ensuring that the assumptions needed to derive the key propositions are satisfied in empirical work. Given this, the index can be used to validate our priors about investors’ behaviour around these events.

4.1 Identification of common shocks

As explained in Section 2, the assumption of independent returns needs to be complemented by a method to identify common shocks in order to ensure that the index is indeed capturing changes in investors’ risk appetite. This is accomplished by computing changes in riskiness on assets in the portfolio at any given time.

Figure 3 reports changes in riskiness on factors in the portfolio at several different points in time. The values are reported in percentage terms, with positive values indicating an increase in riskiness and negative a decrease. Common shocks would correspond to situations where riskiness of all assets has either gone up or down.

Figure 3: Changes in riskiness

In presentation of results we have selected several episodes labelled in Figure 2. None of these periods were characterized by uniform movement in riskiness, although in some
periods majority of changes were in the same direction. September 1992, majority of changes were in direction of increased risk, whereas in October 1987 was characterized by wide spread decreases. Events in February 1985 and October 1997 show very little change.

In applying the above procedure to identify common shocks one has to bear in mind that the procedure is valid only when asset returns are independent, since only in that case does asset volatility coincide with a measure of riskiness of this asset as part of a portfolio. Furthermore, although the number of factors in our portfolio corresponds to the number original assets, factors should not be interpreted as representing individual assets. As stated earlier, each factor is a derivative asset, a linear combination of original assets comprising a portfolio.

4.2 RAI-MI and GRAI

How does the above index compare to the GRAI of Kumar and Persaud (2003)? Figure 4 the smoothed values of both indices using the same underlying assets and procedure proposed by Kumar and Persaud (2003).

Figure 4: GRAI and RAI-MI

Both indexes have similar dynamics over much of the sample, although GRAI is much more volative with $\sigma_{GRAI}^2 = 0.30$, and $\sigma_{RAI-MI}^2 = 0.13$ over the sample period, based on the unsmoothed data. Contemporaneous correlation is 0.62.

Whereas the indices more closely together, especially in the 1990s, there are some differences. For example, in the mid-80s the GRAI indicates a dramatic decrease in investors’ risk appetite, whereas RAI-MI is in the neutral territory. In late 1980s RAI-MI
moves from neutral territory to indicates lower risk appetite. GRAI follows eventually, but indicates that before a decrease in risk appetite there was a sudden and sharp increase.

Recall that the two indices use the same methodology, the only difference being that the RAI-MI is computed on transformed data so that the key condition needed to break observational equivalence holds. The difference is results gives us a sense of the sensitivity of the index to violation of the assumption of independence of returns.

The MI and GRAI suggest risk appetite was generally high in the mid-1980s, but then fell sharply in 1985 with the dramatic realignment of global currencies (the Plaza Accord). Risk appetite was low preceding the stock market crash of 1987 according to the GRAI, while the MI shows a decline beginning with the crash and continuing through to the end of the North American recession in the early 1990s. In the late 1990s the two indexes began to trend higher and move more closely together.

According to both measures, risk appetite peaked in early 1997 during the peak of the corporate bond market. It then fell sharply during the ensuing Asian/Russian/LTCM crises. Appetite rebounded in 1999 and peaked in early 2000 at the height of the stock market bubble. Following the equity sell off in 2000, the indexes declined steadily and hit cyclical lows in late 2002. Despite a general improvement in economic and financial conditions since then, the current level of risk appetite remains relatively low according to both measures.

### 4.3 RAI-MI and 'safe haven' assets

RAI-MI can also be used to assess the indicators of changes in risk appetite typically followed by market practitioners. Whereas they do not rely on a single indicator, some
are typically followed more closely than others. In foreign exchange markets Swiss franc and gold are thoughts to be ‘safe haven’ assets. Decreases in investors’ risk appetite would be reflected in purchases of these assets, which would result in increases of their respective prices.

Our index can be used to examine these indicators. If these assets act as safe haven, one would expect that the periods of low risk appetite, as indicated by RAI-MI, would be the period of high prices of these assets, and the opposite would be the case when risk appetite is high.

Figure 5: RAI-MI and CHF

Figure 6: RAI-MI and gold

Figures 5 and 6 represent the behaviour of the RAI-MI (smoothed version) and the two assets under consideration. In case of CHF, the correlation for the sample period 1981-2005 is 0.49, indicating that, indeed, periods of high risk appetite are associated with low values of this currency and vice versa, a finding consistent with the safe haven story. A regression

\[ CHF_t = a_0 + a_1 RAI-MI_t + e_t \]

yields statistically significant estimate of \( a_1 = 0.71 \) (p value = 0.00), and \( R^2 = 0.24 \).

The case of gold is somewhat less clear. Overall in the full 1981-2005 sample there does not seem to be any trace of a linear relationship between gold and the RAI-MI: the

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12 The results in this section are based on the smoothed values of RAI-MI. The results with nsmeothed RAI-MI are qualitatively similar.
Table 1: Cross-correlations of various indices

correlation is at -0.05, and and the regression

\[ GOLD_t = b_0 + b_1 RAI-MI_t + e_t \]

yields an estimate of \( b_1 \) that is statistically not different from zero. The situation changed rather dramatically in the 1990s. In the period 1990 - 2005, the correlation between gold prices and the RAI-MI is \(-0.45\). The above regression yields \( b_1 = -84.63 \) (p value = 0.00) with \( R^2 = 0.19 \) Thus, over the last 15 years the behaviour of gold is consistent with safe haven explanations used by practitioners.\(^\text{13}\)

### 4.4 Comparison with other indices

In Table 1 we report the values of cross-correlations for a range of indices in current use. The indices included are Goldman Sachs Risk Aversion Index (GC), Credit Suisse First Boston (CSFB), a BIS index proposed by Tarashev et al. (2003), Bank of England index (BE) proposed by Gai and Vause (2004), and the Investor Confidence index (ICI) proposed by Froot and O’Connell. Bold-faced values are significant at the 5 percent level.\(^\text{14}\)

\(^\text{13}\) Of course, the price of gold, as well as any other asset, can move for reasons other than a change in investors’ risk attitudes. The above analysis gives an idea about the explanatory power only for changes in risk attitudes.

\(^\text{14}\) Cf. Illing and Mayer (2005), Table 2. The authors provide details on computation of these indices.
Goldman Sachs Risk Aversion index and the Bank of England index are both highly correlated with RAI-MI, the correlations being 0.73 and 0.63, respectively.

The Goldman Sachs Risk Aversion Index (GS) behaves similarly to RAI-MI even though it employs an entirely different framework for measuring risk appetite. One notable difference between the two series is the much sharper downward spike of the GS in August 1998, the month in which Russia defaulted on its sovereign debt catalyzing the collapse of LTCM.

FIGURE 7: GS and RAI-MI

Similarly, the index suggested by Gai and Vause (2004) behaves similarly to RAI-MI. This index extends the approach of Karampatos, Tarashev, and Tsatsaronis (2003). Perhaps surprisingly the results of this index differ in a marked way from the BIS results.

FIGURE 8: RAI-MI and BE

Of the remaining indices, perhaps the most conspicuous result is that related to the Investor Confidence Index of Froot and O’Connell (2003). Correlation between this and all other indices, except for the BE, are statistically insignificantly different from zero.

Overall, we find that the Bank of England index and the Goldman Sachs index are likely capturing changes in investors risk attitudes, whereas the situation with the remaining three indices is unclear.

15 The GS uses a standard consumption capital asset pricing model where the Arrow-Pratt coefficient of risk aversion is allowed to vary over time (Goldman Sachs, 2003). The model incorporates monthly real US per-capita consumption, the real 3-month US Treasury bill rate, and the inflation-adjusted S&P 500 index. To convert the GS into a risk appetite index one simply multiplies by -1.
5. Conclusions

The profusion of indices purporting to capture changes in investors’ risk appetite and the contradictory signals they offer to investors raises the question which one of them, if any, in fact, captures changes in risk appetite. We build on the work of Kumar and Persaud (2003) and Misina (2003) and propose an index that can separate changes in prices due to changing risk attitudes from changes due to changing asset riskiness. Kumar and Persaud offer an intuitively appealing argument regarding the effects of changes in risk attitudes on asset prices in a portfolio. Misina (2003) establishes the condition under which these effects will indeed be present. The contribution of the this paper is to propose a method that can be applied to any portfolio that would empirically implement the key condition of independent return, and thus validate the interpretation of rank correlation as capturing changes in risk attitudes.

The benchmark proposed here can be used to assess the existing indices of risk appetite, or to validate our priors regarding the behavior of investors’ risk attitudes during particular historical episodes. Furthermore, to the extent that financial crises and flight to liquidity can be attributed to sudden changes in investors’ risk attitudes, the index can be used as an indicator of financial stability in the emerging markets.\(^\text{16}\)

\(^{16}\) IMF has used the version of the index proposed by Kumar and Persaud (2003) – see, for example IMF (2002, 2003).
6. References


7. Appendix A

7.1 Technical details of factor analysis

Starting point of factor analysis is the variance/covariance matrix \( V \), associated with the returns matrix \( R \). The problem is to decompose the information about covariances into its components. This is done by diagonalization of \( V \). Since \( V \) is a real symmetric matrix by construction this task is easy.

**Proposition 7.1 (Lipschutz, 8.14)** Let \( V \) be a real symmetric matrix. Then there exists an orthogonal matrix \( P \) such that the matrix \( D \)

\[
D = P^{-1}VP 
\]

is diagonal.

If we choose for the columns of \( P \) the normalized orthogonal vectors of \( V \), the diagonal entries of \( D \) will be the eigenvalues of \( V \). It also follows that matrices \( D \) and \( V \) are similar.

The next step is to generate factors that correspond to \( D \). This is achieved by a change of coordinates: returns are represented in the new coordinate system associated with \( D \). From

\[
R_t = PF_t 
\]

Note that from the point of view of this procedure it doesn’t matter what the interpretation of the matrix elements is. In our case, it is the covariances, but the analysis is quite general, irrespective of the interpretation of the matrix.
it follows that

$$F_t = P^{-1}R_t, \ \forall t,$$

where $F_t = \begin{bmatrix} F_{1t} & \cdots & F_{Kt} \end{bmatrix}$ This generates a set of factors associated with $D$.

### 7.2 Transformation procedure

The data is given as a series of returns on $k$ assets:

$$R = \begin{bmatrix} R_1 & \cdots & R_K \end{bmatrix},$$

where $R_k$ is a $T \times 1$ vector of returns on asset $k$, $k = 1, \ldots, K$.

The procedure consists of the following steps:

(i) standardize the returns data:

$$R^st = \left( \frac{R - \text{mean}(R)}{\sigma_R} \right)$$

(ii) compute the correlation matrix, $\rho_{R^st}$, for the standardized data

(iii) compute the eigenvalues and eigenvectors associated with the correlation matrix $\rho_{R^st}$. The eigenvectors form an orthogonal basis. The loadings matrix is the matrix of eigenvectors. Denote it by $B$.

(iv) Obtain factor loadings using the fact that at each point in time $t$,

$$\hat{R}(t) = B_{K \times 1} * f(t)$$

so that

$$f(t) = B^{-1}R(t), \ \forall t$$

21
Each factor’s value at time $t$ is its return, which is a linear combination of returns on existing assets. The transformed data are then given by a $T \times K$ matrix

$$F = \begin{bmatrix} F_1 & \ldots & F_K \end{bmatrix},$$

where $F_k = \begin{bmatrix} f(1) & \ldots & f(T) \end{bmatrix}', \forall k.$

## 8. Appendix B: Data

The empirical results were obtained using foreign exchange spot and 3-month forward premiums/discounts. The data is obtained from the Bank for International Settlements, DatastreamTM, and the Bank of Canada. The basket starting in 1983 includes currencies for the following thirteen countries: Austria, Belgium, Canada, Denmark, France, Germany, Great Britain, Italy, Japan, Netherlands, Norway, Sweden, and Switzerland. Following the introduction of the Euro in 1999, the EMU currencies are replaced by the single currency. Currencies for the following countries were added in 1998: Australia, China (Hong Kong), Czech Republic, Hungary, India, South Korea, Malaysia, Mexico, New Zealand, Philippines, Saudi Arabia, Singapore, South Africa, Taiwan, Thailand, Turkey, and United Arab Emirates.
Figure 1
Risk Appetite Index (RAI-MI)
unsmoothed

Figure 2
Risk Appetite Index (RAI-MI)
(Hodrick-Prescott trend of monthly values)
Figure 3: Changes in riskiness of derivative assets

Changes in riskiness of derivative assets in February 1985

Changes in riskiness of derivative assets in October 1987

Changes in riskiness of derivative assets in September 1992

Changes in riskiness of derivative assets in October 1997
Figure 4
RAI-MI and Kumar and Persaud GRAI
(Hodrick-Prescott trend of monthly values)
Figure 5
RAI-MI and Swiss Franc

Figure 6
RAI-MI and gold price
Figure 7
RAI-MI and Goldman Sachs Index

Source: Goldman Sachs

Figure 8
RAI-MI and Gai and Vause (GV) Index

Source: Gai and Vause (2004)