Asymmetries and Portfolio Choice *

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Abstract

Asset returns comove and display asymmetric distributions. Also, investors' attitudes towards risk show an asymmetric treatment of losses versus gains and a tradeoff between their amplitude and frequency. Motivated by these facts, we examine the portfolio choice problem of an investor with generalized disappointment aversion utility, who faces a set of asset returns described by a multivariate extended skew-normal distribution. We show that the proposed setup, for which we develop an analytical solution, can rationalize various forms of equity investing and explain popular portfolio advices that are puzzles to standard models.

Keywords: Downside risk penalty, asymmetry-variance portfolio

JEL Classification: G11

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1 Introduction

Asset returns are predictable, co-move, and display asymmetric distributions with fatter tails than the normal distribution. In addition, correlations between asset returns conditional on downside and upside moves display an asymmetric pattern. In particular, correlations between stocks tend to be much greater for downside moves, especially for extreme downside moves, than for upside moves, as studied empirically by Ang and Chen (2002) and Hong et al. (2006). Also, long-term bonds are negatively correlated with stocks conditional on down markets, and are positively correlated with stocks conditional on up markets.

The aversion to these correlated downside movements between assets, because they may undermine diversification benefits when particularly needed, can be modeled through utility theories that emphasize investors' aversion to downside risk. Such theories include loss aversion demonstrated by Kahneman and Tversky (1979) in their prospect theory of choice, rank-dependent expected utility that emerges from the anticipated utility theory of Quiggin (1982), and the theory of disappointment aversion of Gul (1991) that has recently been generalized by Routledge and Zin (2010). These preferences are consistent with the puzzling experimental behavior observed in the Allais (1979) paradox, as well as with the observation that many people both purchase lottery tickets and insure against losses. They then provide a convenient framework for investigating optimal asset allocation when investors place different weights on downside losses and upside gains.

Our objective is to investigate the joint impact of these two types of asymmetries on portfolio choice: asymmetries in asset returns and asymmetries in investor attitude towards risk. The proposed theoretical setup is simple and parsimonious as it operates in a static setting, explicitly ruling out any effect that might otherwise arise from purely dynamical channels. As we argue throughout the paper, this simple static setting generates portfolio choice behaviors that are consistent with real-life situations as well as asset demands that are consistent with portfolio recommendations by popular financial advisors.

In modeling asymmetric investor preferences, we focus on disappointment aversion utility of Gul (1991) and its generalization put forward by Routledge and Zin (2010). This utility specification is axiomatic, normative and firmly grounded in formal decision theory under uncertainty. In a series of experiments studying decision making under uncertainty of subjects who face a portfolio choice problem or a consumer decision problem, Choi et al. (2007) show that the disappointment aversion utility of Gul (1991) provides a good interpretation of the data at the individual level and can account for the highly heterogeneous behaviors observed in the laboratory. Gill and Prowse (2012) also provide experimental evidence that people are disappointment-averse when they compete, responding negatively to their rival's effort. Such a competition, that may be observed among portfolio managers as studied by Asparouhova et al. (2012) and Basak and Makarov (2013), is a further motivation for examining the optimal asset allocation problem of disappointment-averse investors.

To model asymmetries in asset returns we use the multivariate extended skew-normal distribution developed by Azzalini (1985, 1986) and Adcock and Shutes (2001). The distribution assumes that idiosyncratic security risks follow a joint multivariate gaussian distribution, while skewness is generated by a single common factor that follows a truncated normal distribution, but upon which different securities have different loadings. We demonstrate that the proposed model captures well both the skewness of individual asset returns and the coskewness between assets. Moreover, it is able to match other key features of the return data, like fat tails and asymmetric correlations.

Other authors have looked at portfolio choice under disappointment aversion utility. In particular, we relate to Ang et al. (2005), but extend their analysis in several important directions. First, we consider generalized disappointment aversion utility where the reference point distinguishing disappointing and non-disappointing outcomes may deviate from the certainty equivalent.[1] Second, while Ang et al. (2005) consider a symmetric return distribution, we also study the effect of asymmetric return distributions. Third, our proposed setup easily accommodates multiple risky assets in the portfolio.

Several researchers also examine portfolio choice under return asymmetries. We relate to Das and Uppal (2004) through the assumed asset return distribution, but differ in that our investor has nonstandard preferences and explicitly cares about the downside risk that underlies the asymmetric return distribution. In addition to quantifying the certainty equivalent cost of ignoring returns asymmetry as in Das and Uppal (2004), we can quantify the certainty equivalent cost of ignoring preferences asymmetry. A common approach in the literature for studying the portfolio choice implications of asymmetric returns is to use a third- or a fourth-order Taylor's expansion of a differentiable utility function. Asymmetry aversion of the investor is equivalent to the coefficient of the third-order term in the expansion. This coefficient is then set to a value implied by some standard utility (as in Jondeau and Rockinger; 2006, Guidolin and Timmermann; 2008, or Martellini and Ziemann; 2010), or determined in an ad hoc way (as in Harvey et al.; 2010). In our approach aversion to skewness arises as a result of the investor's loss aversion and we are able to directly characterize the link between them.

We derive an approximate analytical solution to the proposed portfolio choice problem and show that it leads to a three-fund separation as in Simaan (1993). The optimal portfolio

[1] In the original version of the utility by Gul (1991), the reference point is defined as the certainty equivalent of the investment. Ang et al. (2005) study the portfolio choice implications of this original version. Routledge and Zin (2010) introduce a generalized version of the preferences, where the reference point can be set to be different from the certainty equivalent.

is composed of three different funds: the risk-free asset, the standard mean-variance efficient portfolio, and an asymmetry-variance efficient portfolio of the risky assets. The composition of the last fund depends on the skewness of the risky asset returns. The weights assigned by an investor to each of these funds depend primarily on her preference parameters.

The analytical solution to the problem enables us to characterize the effect of asymmetries on the optimal portfolio. If there is no asymmetry in the return distribution, i.e. returns are jointly log-normal, the asymmetry-variance fund becomes redundant. The investor chooses to invest in only two funds, the risk-free asset and the mean-variance efficient portfolio. Disappointment averse investors in this scenario simply increase the weight on the risk-free asset in their optimal portfolio. If, on the other hand, asset returns are asymmetric, the investor will allocate some of her wealth to the asymmetry-variance fund.

Using a calibrated example involving long-term bonds and stocks as risky assets, we demonstrate several implications of the model. First, an investor who is not disappointment averse, allocates a very small fraction of her wealth to the asymmetry-variance fund. In other words, standard symmetric preferences imply that return skewness has only marginal effect on the composition of optimal portfolios. Second, a disappointment averse investor who is sufficiently loss-averse around her certainty equivalent does not hold risky securities, and invests all her wealth in the risk-free asset instead. This is in line with the findings of Ang et al. (2005). Third, a disappointment averse investor whose reference point is lower than the certainty equivalent of the investment always finds it optimal to hold risky assets. Moreover, the asymmetry fund becomes more important in her optimal portfolio. Disappointment aversion induces her to shift from negatively skewed assets to assets with no or positive skewness. Increasing disappointment aversion can cause significant shifts in the composition of the risky portfolio.

When quantifying the cost of ignoring different types of asymmetries, we find that ignoring preference asymmetry (not taking into account that the investor values gains and losses differently) is twice as costly as ignoring return asymmetry (not taking into account that some assets have skewed returns).

We also study the effect of investment horizon on the portfolio choice of disappointment averse investors. When the investment horizon is short, the negative skewness of the risky portfolio (implied by the negative skewness of stocks) deters the investor from holding risky assets. She invests large part of her wealth in the risk-free asset. However, assuming independent return dynamics, asset returns become more symmetric as the investment horizon increases. Consequently, investors with longer investment horizon assign larger weight to risky securities, and shift from bonds to stocks within the risky portfolio.

Finally, we use the proposed setup to address the portfolio allocation puzzle put forward by Canner et al. (1997). Popular financial advisors recommend that more risk-averse investors, who invest in stocks, bonds and cash, should allocate a higher fraction of their risky portfolio (stocks plus bonds) to bonds. Canner et al. (1997) point out that this prediction is inconsistent with the standard portfolio choice theory of Sharpe (1964), Lintner (1965) and Mossin (1966), which implies that all investors should hold the same composition of risky assets, implying a constant bonds/stocks allocation ratio among different type of investors. Canner et al. (1997) explore various possible explanations of this puzzle and find them unsatisfactory.[2] We show that in our setup, non-normality of at least one of the asset log returns is a necessary, but not sufficient condition for explaining the asset allocation puzzle of Canner

[2]Some possible explanations for this divergence between theory and practice have been put forth in the literature. Bajeux-Besnainou et al. (2001) explain the puzzle by assuming that the investor's horizon may exceed the maturity of the cash asset, while Shalit and Yitzhaki (2003) instead use conditional stochastic dominance arguments to illustrate that advisors, acting as agents for numerous clients, recommend portfolios that are not inefficient for all risk averse investors. Campbell and Viceira (2001) rationalize the popular advice in the context of intertemporal asset allocation models with time-varying expected returns.

et al. (1997). Coupled with the generalized disappointment aversion utility of Routledge and Zin (2010) we can rationalize portfolio recommendations by popular financial advisors.

2 Theoretical Setup

We assume that a fund manager, who has generalized disappointment aversion utility as in Routledge and Zin (2010), can allocate her wealth between k risky securities denoted $i = 1, 2, \ldots, k$ and a risk-free asset denoted $i = f$. Similar to papers such as Ang and Bekaert (2002), Das and Uppal (2004), Ang et al. (2005) and Guidolin and Timmermann (2008), we consider a finite-horizon setup with utility defined over terminal wealth. Our proposed model of asset returns is set in discrete time.

2.1 Investor Attitude Towards Risk

The fund manager objective at date 0, starting with the initial fund value F_0, is to maximize the utility of the certainty equivalent of the terminal value F_T at date T. Generalized disappointment aversion preferences (GDA) have the desirable property that investors care differently about downside losses than they care about upside gains. Disappointment averse investors are loss averse around an endogenous reference point that is proportional to their certainty equivalent. Following Routledge and Zin (2010), the utility of the certainty equivalent $\mathcal{R}(F_T)$ of the terminal fund value is implicitly defined by

$$\theta U\left(\mathcal{R}\left(F_T\right)\right) = E\left[U\left(F_T\right)\right] - \ell E\left[\left(U\left(\kappa\mathcal{R}\left(F_T\right)\right) - U\left(F_T\right)\right) I\left(F_T < \kappa\mathcal{R}\left(F_T\right)\right)\right] \qquad (1)$$

where $I(\cdot)$ is an indicator function that takes the value 1 if the condition is met and 0 otherwise, and where

$$U(X) = \frac{X^{-\alpha}}{-\alpha} \quad \text{if } \alpha \neq 0$$
$$= \ln X \quad \text{if } \alpha = 0. \tag{2}$$

The parameter $\alpha \geq -1$ measures the manager's excess risk aversion over the log investor. The parameters $\ell \geq 0$ and $\kappa > 0$ are respectively the manager's loss aversion and the percentage of her certainty equivalent such that outcomes below it are considered disappointing.

The coefficient $\theta \equiv 1 - \ell(\kappa^{-\alpha} - 1) I(\kappa > 1)$ allows to capture both sides of noncentral disappointment in a unique setting. If $\kappa < 1$, the random future value is considered disappointing if it lies sufficiently below today's certainty equivalent. Alternatively, if $\kappa > 1$, the random future value must be sufficiently far above the manager's certainty equivalent to be considered not disappointing. As pointed out by Routledge and Zin (2010), monotonicity imposes the restriction that $\theta > 0$. When ℓ is equal to zero, the manager displays expected utility (EU) preferences, while for $\ell > 0$, outcomes lower than $\kappa\mathcal{R}(F_T)$ receive an extra weight, decreasing the manager's certainty equivalent relative to expected utility. The special case $\kappa = 1$ corresponds the original (central) disappointment aversion preferences of Gul (1991).[3] We subsequently refer to an investor with $\kappa = 1$ as a DA investor and to investors with $\kappa \neq 1$ as GDA investors.

[3] In an intertemporal consumption-based general equilibrium asset pricing model, Routledge and Zin (2010) discuss the value of this parameter in connection with the autocorrelation of consumption growth modeled as a simple two-state Markov chain. In order to generate counter-cyclical risk aversion, they state that a value less than one for κ is needed when there is a negative autocorrelation of consumption growth and a value greater than one when the autocorrelation is positive. The economic mechanism behind this link is the substitution effect.

The terminal fund value may be written

$$F_T = F_0 R_{F,T} \tag{3}$$

where $R_{F,T}$ is the fund gross return over the investment horizon T. Due to the homogeneity of the utility function (2), the certainty equivalent of the terminal fund value is equal to the initial fund value times the certainty equivalent of the fund gross return:

$$\mathcal{R}(F_T) = \mathcal{R}(F_0 R_{F,T}) = F_0 \mathcal{R}(R_{F,T}). \tag{4}$$

Ultimately, the manager's objective is simply to maximize the certainty equivalent of the fund gross return $\mathcal{R}(R_{F,T})$ given by

$$\theta U(\mathcal{R}) = E[U(R_{F,T})] - \ell E[(U(\kappa\mathcal{R}) - U(R_{F,T})) I(R_{F,T} < \kappa\mathcal{R})], \tag{5}$$

where we have used the short-hand notation $\mathcal{R}$ for $\mathcal{R}(R_{F,T})$. This is also equivalent to maximizing its logarithm, $\eta \equiv \ln \mathcal{R}$, subject for example to an asset allocation policy.

Proposition 2.1 *Regardless of the distribution of asset returns, the manager's log certainty equivalent η is implicitly given by*

$$\begin{aligned} \eta &= -\frac{1}{\alpha} \ln E[\exp(-\alpha r_{F,T})] + \frac{1}{\alpha} \ln\left(\theta + \ell\kappa^{-\alpha}(1 - E[\exp(\alpha p_{F,T})])\right) \quad \textit{if } \alpha \neq 0 \\ &= E[r_{F,T}] - \ell(E[p_{F,T}] - \max(\ln\kappa, 0)) \quad \textit{if } \alpha = 0, \end{aligned} \tag{6}$$

where

$$p_{F,T} \equiv \max(\ln\kappa + \eta - r_{F,T}, 0) \tag{7}$$

corresponds to the payoff of a European put option on the fund log return $r_{F,T}$*, with strike equal to* $\ln \kappa + \eta$*, its endogenous threshold of disappointment.*

Proof. See Appendix A.1 for the first case in equation (6). The second cases derives directly from the first cases by taking the limit and applying l'Hôpital's rule.

Equation (6) presumes that the term inside the second log operator in the right-hand side is positive, which is the case indeed.[4] Most importantly, equation (6) shows that the manager's log certainty equivalent is a sum of two components. The first component is the log certainty equivalent corresponding to expected utility with excess risk aversion parameter α, while the second component is non positive by definition, and a decreasing function of the put option payoff. To see the latter, notice that the derivative of the right-hand side of equation (6) with respect to the put option variable is equal to

$$\frac{\partial \eta}{\partial p_F} = -c \quad \text{where} \quad c = \frac{\ell \kappa^{-\alpha} E\left[\exp\left(\alpha p_{F,T}\right)\right]}{\theta + \ell \kappa^{-\alpha}\left(1 - E\left[\exp\left(\alpha p_{F,T}\right)\right]\right)} \geq 0. \tag{8}$$

Notice that the disappointment threshold $\ln \kappa + \eta$ on the fund log return may also be interpretable as the hurdle rate or minimum acceptable return required by the manager. The disappointment-averse manager thus maximizes the hurdle rate and disappointment occurs when the fund log return is smaller than this hurdle rate. Equation (6) thus shows that relative to expected utility, generalized disappointment aversion utility incorporates a downside risk penalty for achieving a fund return that is smaller than the endogenous hurdle rate. The second component in the right-hand side of equation (6) represents the total cost of downside risk. As shown, the shortfall between the fund return and the hurdle rate is valued as a European put option on the fund return with strike price equal to the hurdle

[4] We subsequently derive the closed-form expression of that term when we assume skew-normally distributed log returns and confirm this observation.

rate; the total cost of downside risk is a function of that put option while the coefficient c defined in equation (8) may be interpretable as the marginal cost of downside risk. This means that a one basis point increase in the portfolio shortfall translates into a c basis points decrease in the manager's certainty equivalent. Notice that $c = \ell$ in the special case $\alpha = 0$.

We define the marginal contribution of every asset i to the portfolio return, named *Marginal Reward* (MR), as the partial derivative of the portfolio log return $r_{F,T}$ with respect to the weight of asset i in the portfolio. Similarly, the marginal contribution of asset i to the portfolio shortfall, named *Marginal Shortfall* (MS), is the partial derivative of the portfolio shortfall $p_{F,T}$ with respect to the weight of asset i in the portfolio. Formally, we have

$$MR^i_{F,T} = \frac{\partial r_{F,T}}{\partial w_i} \quad \text{and} \quad MS^i_{F,T} = \frac{\partial p_{F,T}}{\partial w_i}. \tag{9}$$

Proposition 2.2 *The first-order conditions for an optimal asset allocation policy chosen to maximize the log certainty equivalent defined in equation* (6) *are given by*

$$\frac{E^{\mathbf{H}}\left[MR^i_{F,T}\right]}{E^{\mathbf{D}}\left[MS^i_{F,T}\right]} = c, \quad \text{for} \quad i = 1, 2, \ldots, k, \tag{10}$$

where c is given by equation (8) *and where $E^{\mathbf{H}}[\cdot]$ and $E^{\mathbf{D}}[\cdot]$ respectively denote the expectation operators associated with the densities H_T and D_T, defined by*

$$H_T = \frac{\exp(-\alpha r_{F,T})}{E\left[\exp(-\alpha r_{F,T})\right]} \quad \text{and} \quad D_T = \frac{\exp(\alpha p_{F,T})}{E\left[\exp(\alpha p_{F,T})\right]}. \tag{11}$$

Proof. See Appendix A.1.

Equation (10) is intuitive and carries two important pieces of information. First, all assets must have the same risk-adjusted performance, and second, that risk-adjusted performance must be equal to the marginal cost of downside risk. The risk-adjusted performance of an asset is measured as the ratio of its expected marginal reward (adjusted for regular risk) to its expected marginal shortfall (adjusted for downside risk). This ratio looks very familiar with the Omega ratio or the gain-loss ratio of Bernardo and Ledoit (2000) that are common risk-adjusted performance measures in practice. Regular risk is characterized by the covariance with the portfolio log return $r_{F,T}$ as usually understood, which the risk-adjustment density is given by H_T. Similary, downside risk is characterized by the covariance with the portfolio shortfall $p_{F,T}$ which the risk-adjustment density is given by D_T. Observe that in the limiting case $\alpha = 0$, the two risk-adjustment densities reduce to $H_T = D_T = 1$, and the associated expectation operators $E^{\mathbf{H}}[\cdot]$ and $E^{\mathbf{D}}[\cdot]$ reduce to the expectation operator $E[\cdot]$ associated with the physical density. Next, we model asset return dynamics and then derive the optimal allocation.

2.2 Model of Asset Returns

We postulate that the dynamics of log returns on risky assets are well-captured by a multivariate conditional distribution with extended skew-normal marginal densities. A systematic treatment of scalar and multivariate skew-normal distributions can be found in Azzalini (1985, 1986), Azzalini and Dalla Valle (1996) and Gupta et al. (2004). Many extensions of these distributions are surveyed by Azzalini (2005) and some applications to risk measurement, asset pricing, and capital allocation can be found in Vernic (2006), Adcock (2010) and Harvey et al. (2010). To the contrary of these papers, our general setting allows for serial correlation in returns. Formally, log returns on assets may be written

$$r_t = \phi_0 + \phi_1 z_{0,t} + z_t \quad \text{where} \quad z_t = \Phi_z z_{t-1} + u_t, \tag{12}$$

and where the constants ϕ_0 and ϕ_1 are k-dimensional vectors, the scalar $z_{0,t}$ is a skewed shock that is common across all assets and has an independent and identically distributed (IID) standard normal distribution truncated from below at $-\tau$, and the k-dimensional vector u_t is a gaussian white noise process, independent of $z_{0,t}$, and with covariance matrix Ω_u. If $\phi_1 = 0$, then returns are conditionally normal and follow a standard first-order vector autoregressive (VAR) process. The system (12) can be expanded to include other state variables that are potential return predictors, such as in Barberis (2000) and Campbell et al. (2003) among others, and without altering the tractability of the current setting. Also notice that equation (12) is written in a state-space form and can easily be estimated by quasi-maximum likelihood through the Kalman filtering technique. Moreover, explicit closed-form expressions of the unconditional moments of returns exist, alternatively allowing for a generalized method of moments (GMM) estimation with exact moment conditions. We follow this latter approach in the empirical section of the article.

The vector ϕ_1 contains the sensitivity of asset returns to the common skewed shock $z_{0,t}$, and this sensitivity is allowed to vary across assets. The skewness and excess kurtosis of $z_{0,t}$ are positive for all values of τ, and converge to 2 and 6 respectively as τ approaches $-\infty$. Notice that the truncated normal distribution is closely related to the exponential distribution. By taking the limit as τ approaches $-\infty$ and using the following parametrization, $\phi_0 = \varphi_0 - \varphi_1 \tau^2$ and $\phi_1 = -\varphi_1 \tau$, the process $\phi_0 + \phi_1 z_{0,t}$ converges to $\varphi_0 + \varphi_1 e_{0,t}$ where $e_{0,t}$ has an exponential distribution with unitary rate parameter.[5] The exponential distribution is suitable for characterizing the occurrence of extreme events, such as large losses/gains,

[5] In this case, the conditional distribution of asset log returns is multivariate normal-exponential. Adcock and Shutes (2012) present expressions for the multivariate normal-exponential and normal-gamma distributions, discuss their properties and demonstrate that there are relationships between the extended skew-normal distribution and the normal-gamma and normal-exponential distributions. More specifically, it is shown that certain limiting cases of the extended skew-normal are normal-gamma and normal-exponential.

that are unfrequent with a small but significant chance. As shown in Adcock and Shutes (2012), a large negative τ leads to very similar results, at least in empirical applications. In such a case, assets with large negative sensitivities to the common skewed shock $z_{0,t}$ are subject to extreme but unfrequent negative returns, while assets with large positive sensitivities are subject to extreme but unfrequent positive returns. The dynamics (12) suggests that the occurrence of such extreme movements is simultaneous across assets, then may be interpretable as a systemic event. In that sense, our discrete-time asset return dynamics shares the properties of the continuous-time dynamics provided in Das and Uppal (2004).

Given the one-period dynamics (12), we show in Appendix A.2 that for an investment horizon $T \geq 1$, the T-period log returns, say between date 0 and date T, may be written

$$r_{i,T} = \mu_i + \sigma_i \left[\delta_i \varepsilon_{0,T} + \sqrt{1-\delta_i^2} \varepsilon_{i,T} \right], \quad i = 1, 2, \ldots, k \tag{13}$$

with $\sigma_i > 0$, where $\varepsilon_{0,T}$ is a common shock that has a standard normal distribution truncated from below at $-\tau$, and the vector of assets' specific shocks $\varepsilon_T = (\varepsilon_{1,T}, \varepsilon_{2,T}, \ldots, \varepsilon_{k,T})^\top$ is a k-dimensional normal random variable with standardized marginals, independent of $\varepsilon_{0,T}$, with correlation matrix Ψ. Notice that all parameters including τ are now functions of the investment horizon T, which we omit for ease of readability [see Appendix A.2 for details].

The vector $\delta = (\delta_1, \delta_2, \ldots, \delta_k)^\top$ contain specific shape parameters of the distribution of asset returns. Its components all belong to the interval $(-1, 1)$. The scalar τ is the common shape parameter that modulates both the amplitude and the frequency of extreme movements in asset prices; it is equal to zero in the original multivariate skew-normal distribution of Azzalini and Dalla Valle (1996). The parameter Ψ is a correlation matrix that modulates the dependence between asset shocks. The mean m_i, the variance ω_i, the skewness s_i and the excess kurtosis κ_i of each log return $r_{i,T},\ i = 1, 2, \ldots, k$ are given by

$$m_i \equiv E\left[r_{i,T}\right] = \mu_i + (\sigma_i \delta_i)\, \xi_1(\tau) \quad \text{and} \quad \omega_i \equiv Var\left[r_{i,T}\right] = \sigma_i^2 + (\sigma_i \delta_i)^2\, \xi_2(\tau)$$
$$s_i \equiv Skew\left[r_{i,T}\right] = \frac{(\sigma_i \delta_i)^3\, \xi_3(\tau)}{\omega_i^{3/2}} \quad \text{and} \quad \kappa_i \equiv XKurt\left[r_{i,T}\right] = \frac{(\sigma_i \delta_i)^4\, \xi_4(\tau)}{\omega_i^2}, \tag{14}$$

where the functions $\xi_j(x),\ j = 1, 2, \ldots$ are the jth order derivatives of the log standard normal cumulative distribution function. If $\delta_i = 0$, then $r_{i,T}$ is normally distributed with mean μ_i and variance σ_i^2. In consequence, δ_i may be interpretable as a skewness parameter or more precisely a parameter of non-normality.

Notice that the third-order moment of $r_{i,T}$ is given by $E\left[(r_{i,T} - m_i)^3\right] = (\sigma_i \delta_i)^3\, \xi_3(\tau)$. More generally, the jth-order cumulant of $r_{i,T}$ is equal to $(\sigma_i \delta_i)^j\, \xi_j(\tau)$ for $j \geq 3$. It turns out that in the class of random variables defined by equation (13), the quantity $(\sigma_i \delta_i)$ fully characterizes the asymmetry in particular, and more generally the non-normality of returns. In consequence, let $Asy[\cdot]$ denotes the asymmetry operator defined by $Asy\left[r_{i,T}\right] = (\sigma_i \delta_i)$. We show that for $u \in \mathbb{R}$ and $v \in \mathbb{R}^k$, $u + v^\top r_T$ may be expressed in the form of equation (13), and that the asymmetry operator satisfies the nice property $Asy\left[u + v^\top r_T\right] = v^\top Asy\left[r_T\right]$ where the asymmetry of a vector random variable is the vector of asymmetries of its scalar components. Notice that the measure of asymmetry or non-normality $a_i \equiv Asy\left[r_{i,T}\right]$ is, up to a dimensionless multiplicative constant, equal to the cubic-root of the third-order moment of returns. Interestingly, the measure of return asymmetry a_i then expresses in the same unit as the mean m_i and the volatility $\sqrt{\omega_i}$. Also observe from equation (13) that $(\sigma_i \delta_i)$ may also be interpretable as the exposure (or covariance) of the asset log return to a market factor whose innovations are driven by the common skewed shock $\varepsilon_{0,T}$.

2.3 Asset Allocation and Endogenous Risk Aversion

The second-order Taylor's approximation *a la* Campbell and Viceira (2002) of the fund log return implies that it may be written

$$r_{F,T} = w^{\top}\left(r_T - \iota r_f + \frac{1}{2}\omega\right) - \frac{1}{2}w^{\top}\Omega w + r_f \tag{15}$$

where w is the vector of portfolio weights on risky assets, ι is a vector of ones, the vector ω is the diagonal of the covariance matrix $\Omega = E\left[(r_T - m)(r_T - m)^{\top}\right]$ and r_f is the yield on a T-period zero-coupon risk-free bond. The fund log return then has a mean m_F, a variance ω_F and an asymmetry a_F given by

$$m_F = w^{\top}\left(m - r_f\iota + \frac{1}{2}\omega\right) - \frac{1}{2}w^{\top}\Omega w + r_f, \quad \omega_F = w^{\top}\Omega w \quad \text{and} \quad a_F = w^{\top}a \tag{16}$$

where $a = Asy\left[r_T\right]$ is the vector of assets' return asymmetries. We also show that, similar to individual assets, the fund log return may be written

$$r_{F,T} = \mu_F + \sigma_F\left[\delta_F\varepsilon_{0,T} + \sqrt{1-\delta_F^2}\varepsilon_{F,T}\right], \tag{17}$$

where

$$\begin{aligned} &\mu_F = w^{\top}\left(\mu - \iota r_f + \frac{1}{2}\omega\right) - \frac{1}{2}w^{\top}\Omega w + r_f \quad \text{and} \quad \sigma_F^2 = w^{\top}\left((\Lambda\Psi\Lambda) + aa^{\top}\right)w \\ &\delta_F = \frac{w^{\top}a}{\sigma_F} \quad \text{and} \quad \varepsilon_{F,T} = \frac{w^{\top}\Lambda\varepsilon_T}{\sqrt{w^{\top}(\Lambda\Psi\Lambda)w}}, \end{aligned} \tag{18}$$

with $\lambda_i = \sigma_i\sqrt{1-\delta_i^2}$ for $i = 1, 2, \ldots, k$ and $\Lambda = Diag(\lambda_1, \lambda_2, \ldots, \lambda_k)$ is a diagonal matrix.

Proposition 2.3 *The solution to the fund manager's optimal asset allocation problem may* *be written as follows:*

$$w = \frac{1}{\gamma}\left(w^{\mathbf{MV}} + \chi w^{\mathbf{AV}}\right) \tag{19}$$

where

$$w^{\mathbf{MV}} = \Omega^{-1}\left(m - r_f\iota + \frac{1}{2}\omega\right) \quad \text{and} \quad w^{\mathbf{AV}} = \Omega^{-1}a, \tag{20}$$

and where analytical expressions of the coefficients γ and χ are given in Appendix A.3.

Proof. See Appendix A.3.

The coefficients γ and χ depend explicitly on η as well as on two sets of parameters: manager's preference parameters α, ℓ and κ, and parameters μ_F, σ_F and δ_F of the distribution of the fund log return. This latter set of parameters depends explicitly on the asset allocation chosen by the fund manager. Thus, if η were known, equation (19) implicitly defines both the optimal asset allocation and the coefficients γ and χ, making these coefficients endogenous to the model. However, η is itself a function of the optimal allocation w and hence, equations (19) must be solved simultaneously with equation (6) that defines η. In the appendix, we also provide an analytical expression of the marginal cost of downside risk c. Similar to the coefficients γ and χ, the marginal cost of downside risk c also depends on the asset allocation chosen by the fund manager, and thus is endogenous to the model.

Interestingly, given the endogenous values of γ and χ, the optimal allocation (19) can also be achieved by solving the following (mean-variance-asymmetry) investment problem:

$$\max_{w} \quad \left(m_F - r_f + \frac{1}{2}\omega_F\right) - \frac{\gamma}{2}\omega_F + \chi a_F \tag{21}$$

where m_F, ω_F and a_F are the mean, the variance and the asymmetry of the fund log return given in equation (16). It is then straightforward to interpret the coefficient γ as the effective risk aversion and the coefficient χ as the manager's implicit asymmetry aversion. This result is appealing. In fact, it shows that effective risk aversion is endogenous under disappointment aversion preferences, consistent with the discussion in Routledge and Zin (2010) and Bonomo et al. (2011) within an intertemporal consumption-based general equilibrium model. However, to the contrary of these authors, we explicitly derive the formula of effective risk aversion and can quantify it in our partial equilibrium setting.[6] In particular, the formula of the coefficient γ given in Appendix A.3 shows that if $\ell = 0$, that is if the fund manager has expected utility preferences, then γ reduces to $1 + \alpha$, the curvature of the power utility function (2) which also measures effective risk aversion under expected utility preferences.

Following Das and Uppal (2004) we quantify the certainty equivalent cost of ignoring returns asymmetry, and we apply the same approach to evaluate the cost of ignoring preferences asymmetry. The log certainty equivalent of the manager is $\eta \equiv \ln \mathcal{R}$. A manager who ignores asymmetry in the distribution of log asset returns and resolves his optimal portfolio choice problem as if log asset returns were normally distributed with same mean and variance-covariance matrix as the true distribution would chose an allocation $\hat{w}$. That suboptimal allocation corresponds to a log certainty equivalent $\hat{\eta} \equiv \ln \hat{\mathcal{R}}$ under the true distribution. The difference $\hat{\mathcal{R}} - \mathcal{R}$ is negative and represents the certainty equivalent cost of ignoring returns asymmetry. Similarly, a manager who ignores asymmetry in preferences and resolves the optimal portfolio choice problem as if preferences were from expected utility with equal effective risk aversion as the true preferences would chose an allocation $\tilde{w}$. That suboptimal allocation corresponds to a log certainty equivalent $\tilde{\eta} \equiv \ln \tilde{\mathcal{R}}$ under the true preferences. The difference $\tilde{\mathcal{R}} - \mathcal{R}$ is negative and represents the cost of ignoring preferences

[6] Bonomo et al. (2011) assess the level of effective risk aversion implied by disappointment aversion preferences by drawing indifference curves for a hypothetical gamble with two equiprobable outcomes, and comparing their curvatures for disappointment-aversion utility and expected utility.

asymmetry. Ignoring preferences asymmetry may occur in delegated portfolio management or in portfolio advice for example if the portfolio manager or the financial advisor can hardly assess the true preferences of the investor or of the client she is acting on behalf of.

Finally, notice that standard solutions to portfolio choice problems with higher-order moments involve a third- or a fourth-order Taylor's expansion of a differentiable utility function as for example in Jondeau and Rockinger (2006), Guidolin and Timmermann (2008) or Martellini and Ziemann (2010). In these studies, the asymmetry aversion is equivalent to the coefficient of the third-order moment of the portfolio return in an objective function equivalent to equation (21). To make our implicit asymmetry aversion comparable to such a measure, we need to divide the coefficient χ by the quantity $a_F^2 \xi_3 (\tau)$ since the third-order moment of the portfolio return $r_{F,T}$ is given by $E\left[(r_{F,T} - m_F)^3\right] = a_F^3 \xi_3 (\tau)$. In all what follows, the reported values of the implicit asymmetry aversion coefficient correspond to this normalized value of the coefficient χ.

3 Empirical Application

3.1 Data and Parameter Estimation

In this section, we use the general theoretical framework for optimal portfolio choice described in Section 2 to investigate how investors who differ in their risk aversion and disappointment aversion preferences allocate their portfolios among three assets: stock, bond and cash. These three assets represent the most familiar portfolio building blocks. For simplicity and to better isolate the impact of the two different forms of asymmetries on portfolio choice, we assume independent and identically distributed (IID) asset log returns, thus $\Phi_z = 0$ in equation (12). Our calibration exercise is based on post-war monthly data for the U.S. stock market, from

January 1963 to December 2012, which we obtain from the Center for Research in Security Prices (CRSP). We construct the excess log stock return as the difference between the log return on the stock market index and the log return on the 30-day Treasury bill. Similarly, the excess log bond return is the difference between the log return on the 30-year government bond index and the log return on the 30-day Treasury bill. The constant risk-free rate is the sample average of the log return on the 30-day Treasury bill. The stock return is the value-weighted return, including dividends, on the NYSE, NASDAQ, and AMEX markets. The 30-year government bond return comes from the U.S. Treasury and Inflation Series file in CRSP. The source of the 30-day bill rate is the CRSP Fama Risk-Free Rates file.

Given the constant one-month risk-free rate $r_f = 0.38\%$ and fixing $\tau = -4$ throughout the paper, Table 1 shows estimation results of excess log return dynamics of the two risky assets, written under the form of equation (13), where the seven one-month model parameters are chosen to minimize the distance between model-implied moments and their sample counterparts, using the generalized method of moments (GMM) with identity weighting matrix. The table displays three different GMM estimation results. First, GMM A is identified and fits the two means, the two volatilities, the correlation and the two skewness, where subscript "S" is used for stock and subscript "B" for bond. Next, GMM B is overidentified, fitting the two coskewness in addition to the seven moments considered in GMM A. Finally, GMM C is similar to GMM A, fitting the same moments but the skewness of bond which is replaced by the coskewness of stock relative to bond. The top panel of the table shows sample moments and fitted moments, while the bottom panel shows model parameter estimates. Notice that for horizons longer than one month, T-month model parameters can be derived from one-month parameters as described in Appendix A.2.

Sample moments reported in Table 1 show that the monthly excess log stock return has a mean of 0.46%, a volatility of 4.26% and a negative skewness of -0.6448, while the monthly excess log bond return has a mean of 0.13%, a volatility of 2.12% and a positive skewness of 0.2019. The monthly correlation between the two risky assets is 0.1046. As shown in the column GMM A, all sample moment estimates are significantly different from zero at conventional levels of confidence, except for the skewness of bond. The same holds in the column GMM B, while in the column GMM C where the skewness of bond is now excluded from the choice of moments compared to GMM B, all seven sample moment estimates are significantly different from zero at conventional levels of confidence, and the model-implied coskewness of bond relative to stock matches well its data counterpart. GMM C is then considered as our preferred parameters configuration, which is subsequently used to calibrate the asset return dynamics.

The proposed multivariate extended skew-normal model of asset returns captures well all key asset return moments, then providing a simple and more reasonable characterization of IID return dynamics. To the contrary, a multivariate normal model of asset returns would assume in particular that the stock skewness and the two coskewness are equal to zero while the data evidence that these higher moments are significantly different from zero at conventional levels of confidence. To further explore the ability of the multivariate extended skew-normal model in matching key features of the asset return data, we use the GMM parameter estimates in Table 1 to compute via simulations the model-implied correlation between bond and stock conditional to stock falling below a given quantile of its distribution, as well as stock expected shortfall at a given quantile of stock distribution. These two quantities are plotted in Figure 1, in Panel A and Panel B respectively, together with their data counterpart and their analogue computed from the normal distribution. Figure 1 confirms that the multivariate extended skew-normal model fits these features of the data far better than the multivariate normal model, and that the GMM C fit is the closest to the data, corroborating our choice of this parameters scenario for the calibration assessment of our portfolio choice problem carried out in subsequent sections.

3.2 Model Calibration and Results

We use GMM C parameters scenario to compute optimal portfolio rules for a value of excess risk aversion $\alpha = 1$, for 1,000 equally spaced values of loss aversion $\ell \in [0, 5]$, and for nine regularly spaced values of the relative disappointment threshold, $\kappa \in [0.96, 1.04]$. We report results for a one-month investment horizon ($T = 1$) in Tables 2 to 4, where the value of loss aversion is set to $\ell = 2$. The reported results, which are displayed both for the GDA investor and for the EU investor with equal coefficient of effective risk aversion γ, provide an exhaustive characterization for the impact of asymmetries on portfolio choice as examined in the current article. We later analyze portfolio implications when the investment horizon varies. In each table, the first panel displays the effective risk aversion coefficient γ, the implicit asymmetry aversion coefficient χ and the marginal cost of downside risk c, in order to ease comparison between GDA and EU investors.

Given the estimated asset returns distribution, the mean-variance efficient portfolio $w^{\mathbf{MV}}$ and the asymmetry-variance efficient portfolio $w^{\mathbf{AV}}$ are given by

$$w^{\mathbf{MV}} = \begin{pmatrix} 2.9055 \\ 2.6108 \end{pmatrix} \quad \text{and} \quad w^{\mathbf{AV}} = \begin{pmatrix} -81.0343 \\ 66.7031 \end{pmatrix} \tag{22}$$

and their normalized counterparts in percentage values are given by

$$\bar{w}^{\mathbf{MV}} = \begin{pmatrix} 52.67\% \\ 47.33\% \end{pmatrix} \quad \text{and} \quad \bar{w}^{\mathbf{AV}} = \begin{pmatrix} 565.44\% \\ -465.44\% \end{pmatrix}. \tag{23}$$

Notice that the sum of portfolio weights in $w^{\mathbf{AV}}$ is negative and that $w^{\mathbf{AV}}$ has a positive asymmetry. To the contrary, the sum of weights in the normalized asymmetry-variance portfolio $\bar{w}^{\mathbf{AV}}$ is positive and it has a negative asymmetry. This means that the asymmetry-variance efficient portfolio is a short position in $\bar{w}^{\mathbf{AV}}$.

3.2.1 Investors' Attitude Towards Risk

There are three main observations from the first panel of Table 2. First, effective risk aversion is infinite for our base case DA investor with $\kappa = 1$. This corroborates the findings by Ang et al. (2005) that DA investors with high enough level of loss aversion do not hold risky securities. Our base case value of loss aversion $\ell = 2$ is larger than the threshold value of $\ell^* = 0.43$ above which investors with one-month horizon do not hold stock and bond. The value ℓ^* is entirely determined by the asset returns distribution. We discuss the effect of loss aversion on investor's attitude towards risk and on asset allocation in Section 3.2.4.

Second, the implicit asymmetry aversion coefficient is positive for GDA investors with $\kappa < 1$ and negative for those with $\kappa > 1$. To understand the rationale behind this result, refer to the second and the third panels of Table 2 that show the endogenous hurdle rate of these two different categories of GDA investors and the marginal expected shortfall of different assets. As a recall, the marginal expected shortfall measures how a position taken in an asset adds to the portfolio's overall risk. In words, it measures the asset's losses when the portfolio as a whole is doing relatively poorly. Similarly, we define the marginal upside potential as a measure of how a position taken in an asset adds to the portfolio's overall reward. In words, it measures the asset's gains when the portfolio as a whole is doing relatively well. The hurdle rate of our base case DA investor is the risk-free rate equal to 0.38%.

The hurdle rate of GDA investors with $\kappa < 1$ is lower than the risk-free rate and can be negative as such investors are only trying to avoid relatively big losses. For example, our base case GDA investor with $\kappa = 0.97$ has an implicit asymmetry aversion coefficient of 625, a certainty equivalent of 0.53% and a hurdle rate of -2.51%. She will be disappointed only if the monthly portfolio log return falls below -2.51%; there are 2.18% chances that this happens, and her portfolio excess return in these worst cases averages to -3.53%. The marginal contribution of stock to this portfolio expected shortfall is -9.86% and is more than four times higher than the marginal contribution of bond that equals -2.30%.

To the contrary, the hurdle rate of GDA investors with $\kappa > 1$ is higher than the risk-free rate as such investors are only chasing relatively big gains. For instance, our base case GDA investor with $\kappa = 1.02$ has an implicit asymmetry aversion coefficient of -260, a certainty equivalent of 0.53% but a hurdle rate of 2.51%. It takes a monthly portfolio log return higher than 2.51% for her not to be disappointed; there are 9.41% chances that this happens, and her portfolio excess return in these best cases averages to 2.77%. The marginal contribution of stock to this portfolio upside potential is 7.03% and is more than three times higher than the marginal contribution of bond that equals 2.01%.

Coming back to the rationale behind different signs of the implicit asymmetry aversion coefficient, start with the mean-variance efficient portfolio $w^{\mathbf{MV}}$ which has a long position in both stock and bond. Compared to a mean-variance investor with equal effective risk aversion, GDA investors with $\kappa < 1$, in order to hedge against large downside losses, reduce their positions in assets with larger contributions to the portfolio expected shortfall upon realizations below the hurdle rate. In general, these assets with relatively large marginal expected shortfall, like stock in our setting, will also display relatively large negative skewness and consequently negative weights in the asymmetry-variance portfolio $w^{\mathbf{AV}}$. Reduced positions in these assets compared to the mean-variance investor imply from equation (19) that the implicit asymmetry aversion coefficient χ is positive for GDA investors with $\kappa < 1$.

Betting on large upside gains, GDA investors with $\kappa > 1$ have an oppositive behavior. Compared to a mean-variance investor with equal effective risk aversion, they take more increased positions in assets with larger contributions to the portfolio upside potential upon realizations above the hurdle rate. These assets with relatively large marginal upside potential, like stock in our setting, generally turn out to be the same assets that have relatively large left-tailed risk. Since stock also has a relatively large negative skewness and consequently a negative weight in the asymmetry-variance portfolio $w^{\mathbf{AV}}$, increased positions in stock compared to the mean-variance investor imply from equation (19) that the implicit asymmetry aversion coefficient χ is negative for GDA investors with $\kappa > 1$. Thus, by liking stock because it has a large marginal upside potential, GDA investors with $\kappa > 1$ turn out implicitly liking negative skewness. Finally notice from our discussion that the contrast between GDA investors with $\kappa < 1$ and GDA investors with $\kappa > 1$ may otherwise be connected with the contrast between hedgers and speculators in real-life situations.

The third and last observation from the first panel of Table 2 is that EU investors have positive implicit asymmetry aversion and that the magnitude of implicit asymmetry aversion is much larger for GDA investors than for EU investors with equal effective risk aversion. For example, the implicit asymmetry aversion coefficient of our base case GDA investor with $\kappa = 0.97$ is 625, about twenty-three times higher than that of her EU counterpart which is only 27. Similarly, the implicit asymmetry aversion coefficient of our base case GDA investor with $\kappa = 1.02$ is -260, which the magnitude is about five times higher than that of her EU counterpart which is only 49. We subsequently show that these differences in sign and magnitude of the implicit asymmetry aversion coefficient among these three categories of investors (GDA with $\kappa < 1$, GDA with $\kappa > 1$, and EU) have important implications for asset demands.

3.2.2 Optimal Asset Allocation

The first panel of Table 3 shows asset demands in stock, bond and cash. The total demands in stock and bond are also decomposed into their mean-variance and asymmetry-variance components. Following our previous discussion, the asymmetry-variance component may well be interpreted either as a hedging demand against big downside losses for GDA investors with $\kappa < 1$, or a speculative demand on big upside gains for GDA investors with $\kappa > 1$. In our setting, investors involved into these hedging and speculative strategies are mainly concerned with their positions in stock as stock is the main and largest contributor both to portfolio left-tailed risk and portfolio right-tailed reward.

Consistent with our previous discussion, the asymmetry-variance demand of stock is negative for GDA investors with $\kappa < 1$ since stock contributes largely to the portfolio left-tailed risk. For instance, our base case GDA investor with $\kappa = 0.97$ has a total demand of 26.58% in stock, out of which 35.66% account for the mean-variance component and -9.07% represent the hedging component. This reduction of 9.07% in the stock position relative to the mean-variance investor is reallocated to increase both the bond position by 7.47% from 32.04% to 39.51%, and the cash position by 1.60% from 32.30% to 33.91%. To the contrary, the asymmetry-variance demand of stock is positive for GDA investors with $\kappa > 1$ since stock contributes largely to the portfolio right-tailed reward. For instance, our base case

GDA investor with $\kappa = 1.02$ has a total demand of 33.98% in stock, out of which 27.27% represent the mean-variance component and 6.71% account for the speculative demand. This increase of 6.71% in the stock position relative to the mean-variance investor is obtained by reducing both the bond position by 5.52% from 24.50% to 18.98%, and the cash position by 1.19% from 48.23% to 47.04%.

Focusing on the total demand in risky securities, the first panel of Table 3 shows that both stock and bond total demands increase for GDA investors as κ decreases from one towards zero. This is because the set of disappointing outcomes gets smaller and GDA investors can afford more larger losses as their hurdle rate is set to more negative values far below their certainty equivalent, as shown in the second panel of Table 2. This tolerance for more larger losses results in an increased risk taking. For example, as κ decreases from 0.99 to 0.96, the hurdle rate decreases from -0.57% to -3.51%, leading to an increase in the total stock demand from 9.22% to 34.80%, while the total bond demand rises from 14.03% to 51.02%. Also, as κ decreases from one towards zero, GDA investors are disappointed only with more left-tailer losses which become more unfrequent but more larger in size, then requiring a more larger stock hedging demand. For instance, the stock hedging demand increases in magnitude from -3.34% to -11.47% as the parameter κ decreases from 0.99 to 0.96, and is reallocated to increase bond and cash positions relative to the mean-variance investor.

The first panel of Table 3 also reveals that both stock and bond total demands increase for GDA investors as κ increases from one towards infinity. This is because GDA investors have a more larger hurdle rate far above their certainty equivalent as shown in the second panel of Table 2, betting on more right-tailer reward that can only be attainable with an increased risk taking. For example, as κ increases from 1.01 to 1.04, the hurdle rate increases from 1.46% to 4.56%, leading to an increase in the total stock demand from 18.19% to 60.06%, while the total bond demand rises from 9.52% to 37.45%. Also, as κ increases from one towards infinity, for GDA investors not to be disappointed, it takes only more right-tailer gains which become more unfrequent but more larger in size, then requiring a more larger stock speculative demand. For instance, the stock speculative demand increases from 3.96% to 9.59% as the parameter κ increases from 1.01 to 1.04, and obtains from reductions in bond and cash positions relative to the mean-variance investor.

As shown in equation (19), investing in cash and risky securities is equivalent to investing in cash and holding each of the two-mutual funds $\bar{w}^{\mathbf{MV}}$ and $\bar{w}^{\mathbf{AV}}$ composed of these risky securities. Proposition 2.3 is thus equivalent to a three-fund separation theorem. Allocations in the two-mutual funds $\bar{w}^{\mathbf{MV}}$ and $\bar{w}^{\mathbf{AV}}$ correspond to the rows $\overline{MV}$ and $\overline{AV}$ of Table 3. Notice that $\overline{MV}$ is identical for all investors with equal effective risk aversion and that $\overline{AV}$ is also equal to the cash allocation of the mean-variance investor in excess of that of the other investor with equal effective risk aversion.

An interesting observation from the first panel of Table 3 is that at equal effective risk aversion, GDA investors and EU investors have very different asymmetry-variance components, a direct consequence of equation (19). As EU investors have a positive implicit asymmetry aversion coefficient, they short the mutual fund $\bar{w}^{\mathbf{AV}}$ just like GDA investors with $\kappa < 1$, but have very small positions in magnitude compared to that of GDA investors with equal effective risk aversion. As an illustration, the $\overline{AV}$ position of our base case GDA investor with $\kappa = 0.97$ is -1.60%, that is more than ten times higher that of her EU counterpart. Also, the $\overline{AV}$ position of our base case GDA investor with $\kappa = 1.02$ is 1.19%, also about ten times higher in magnitude compared to that of her EU counterpart.

Notice that if asymmetries in stock and bond log returns are all assumed equal to zero, implying $w^{\mathbf{AV}} = 0$, then all investors hold the same composition of stock and bond which is determined by the mean-variance efficient portfolio $\bar{w}^{\mathbf{AV}}$, and therefore the bond/stock allocation ratio will be invariant to the investor's attitude towards risk. This shows in our static setting that the non-normality of at least one of the asset log returns is a necessary condition for explaining the asset allocation puzzle pioneered by Canner et al. (1997). The multivariate extended skew-normal assumption for the distribution of log returns then provides an ideal building block for revisiting this asset allocation puzzle in a static setting.

At fixed investment horizon, there exists a unique pair (γ, χ) defining the risk attitude of an investor whose optimal demand is characterized by a given pair $(w_S, w_B/w_S)$ of stock demand and bond/stock allocation ratio. Formally, we find from equation (19) that

$$\chi = -\frac{\left(w_S^{\mathbf{MV}}/w_S\right) - \left(w_B^{\mathbf{MV}}/w_B\right)}{\left(w_S^{\mathbf{AV}}/w_S\right) - \left(w_B^{\mathbf{AV}}/w_B\right)} \quad \text{and} \quad \gamma = \left(w_S^{\mathbf{MV}}/w_S\right) + \chi \left(w_S^{\mathbf{AV}}/w_S\right). \tag{24}$$

Table 5 adapted from Canner et al. (1997) shows the recommendations of four financial advisors to the general public. Taking each of these recommendations as an optimal portfolio choice and using our calibration for cash, bond and stock returns, we can compute the effective risk aversion and the implicit asymmetry aversion of conservative, moderate and aggressive investors as perceived by the four financial advisors. These coefficients are invariant to the time horizon as our asset returns dynamics is IID. Since the recommendations themselves do not specify the time horizon of the investment, there are reasons to believe that they assume or are derived from an IID model. The main observation from Table 5 is that the level of effective risk aversion increases from conservative to aggressive investors as predicted, while the implicit asymmetry aversion coefficient decreases from positive to negative values consistent with our discussion in Section 3.2.1.

The bond/stock allocation ratio of the mean-variance investor is 0.899 with our calibration. We have shown that GDA investors with $\kappa < 1$ and EU investors, because of their positive implicit asymmetry aversion coefficient and the respective negative and positive positions in stock and bond in the asymmetry-variance efficient portfolio, reduce their stock demand and increase their bond demand relative to the mean-variance investor. This has the effect of increasing the bond/stock allocation ratio relative to the mean-variance investor. The effect is more pronounce for GDA investors with $\kappa < 1$ than for EU investors, due to the large difference in their implicit asymmetry aversion coefficient. Our base case GDA investors with $\kappa < 1$ have a bond/stock allocation ratio around 1.50 while EU investors have a bond/stock allocation ratio around 0.94. To the contrary, GDA investors with $\kappa > 1$, because of their negative implicit asymmetry aversion coefficient, increase their stock demand and decrease their bond demand relative to the mean-variance investor, and this has the effect of decreasing the bond/stock allocation ratio relative to the mean-variance investor. Our base case GDA investors with $\kappa > 1$ have a bond/stock allocation ratio around 0.60.

The second panel of Table 3 displays the moments of the portfolio excess return for different types of investors. The main observation is that GDA investors with $\kappa < 1$ choose portfolios with lower expected excess return, lower volatility, lower negative skewness and lower excess kurtosis than their EU counterparts, and that to the contrary, GDA investors with $\kappa > 1$ choose portfolios with higher expected excess return, higher volatility, higher negative skewness and higher excess kurtosis than their EU counterparts. This is the direct consequence of the more conservative investment in stock of the GDA investors with $\kappa < 1$ due to their large positive implicit asymmetry aversion coefficient, and of the more aggressive investment in stock of GDA investors with $\kappa > 1$ due to their large negative implicit asymmetry aversion coefficient. EU investors stand in the middle of these two categories, and can be considered as moderate investors in stock at equal effective risk aversion.

Finally, Table 4 evaluates portfolio expected shortfall and upside potential of different types of investors on the same basis. The main observation is that, regardless of the threshold used to compute expected shortfall and upside potential, GDA investors with $\kappa < 1$ choose portfolios with lower left-tailed risk and lower right-tailed reward than their EU counterparts, while GDA investors with $\kappa > 1$ to the contrary, choose portfolios with higher left-tailed risk and higher right-tailed reward than their EU counterparts. This again reflects differences in levels of implicit asymmetry aversion among these categories of investors. Also observe that stock remains the asset with the largest marginal contribution to both portfolio left-tailed risk and right-tailed reward across different types of investors.

3.2.3 Suboptimal Portfolio Shifts and Costs of Ignoring Asymmetries

A corollary to Proposition 2.3 is that an investor who ignores returns asymmetry ends up behaving like a mean-variance investor. As we have already discussed, in our setting, GDA investors are mainly concerned with their positions in stock as stock is the main and largest contributor both to portfolio left-tailed risk and portfolio right-tailed reward. GDA investors with $\kappa < 1$ who ignore returns asymmetry behave like a mean-variance investor with higher effective risk aversion $\hat{\gamma} > \gamma$, since their willingness to reduce portfolio left-tailed risk implies that they have a lower position in stock relative to the mean-variance investor with equal effective risk aversion. Behaving like a mean-variance investor with higher effective risk aversion also implies that they considerably reduce their bond demand and reallocate to stock and cash relative to the optimal allocation. In consequence, the suboptimal stock and cash demands are higher and the suboptimal bond demand is lower than the optimal.

To the contrary, GDA investors with $\kappa > 1$ who ignore returns asymmetry behave like a mean-variance investor with lower effective risk aversion $\hat{\gamma} < \gamma$, since their willingness to increase portfolio right-tailed reward implies that they have a higher position in stock relative to the mean-variance investor with equal effective risk aversion. Behaving like a mean-variance investor with lower effective risk aversion also implies that they considerably increase their bond demand by reallocating from stock and cash relative to the optimal allocation. In consequence, the suboptimal stock and cash demands are lower and the suboptimal bond demand is higher than their optimal counterparts.

To illustrate these intuitions, the first panel of Table 6 shows differences between suboptimal and optimal demands in stock, bond and cash across different types of GDA investors, as well as the certainty equivalent cost for ignoring returns asymmetry. For instance, our base case GDA investor with $\kappa = 0.97$ who has an effective risk aversion $\gamma = 8.15$ behaves like a mean-variance investor with a higher risk aversion $\hat{\gamma} = 9.05$ when she ignores asymmetry in the distribution of asset log returns, and then suboptimally invests 5.52% more in stock, 5.15% more in cash and 10.67% less in bond relative to the optimal allocation. To the contrary, our base case GDA investor with $\kappa = 1.02$ who has an effective risk aversion $\gamma = 10.66$ behaves like a mean-variance investor with a lower risk aversion $\hat{\gamma} = 9.81$ and instead suboptimally invests 4.35% less in stock, 3.29% less in cash and 7.64% more in bond relative to the optimal allocation when he ignores returns asymmetry. In both cases as more generally for other GDA investors shown in the table, the certainty equivalent costs for ignoring returns asymmetry is very small and represents less than one basis point. This cost for ignoring returns asymmetry is more important for high risk tolerance investors for whom suboptimal shifts in asset demands are also more important. This result corroborates a similar finding by Das and Uppal (2004) on the certainty equivalent costs of ignoring returns asymmetry by EU investors.

The second panel of Table 6 displays suboptimal shifts in asset demands as well as the certainty equivalent cost for ignoring preferences asymmetry. Following up on our previous comparisons between GDA investors and their equivalent EU counterparts with equal effective risk aversion, it is shown that ignoring preferences asymmetry of GDA investors with $\kappa < 1$ results in a larger suboptimal stock demand that is compensated with lower suboptimal demands in cash and bond, while it is the opposite for GDA investors with $\kappa > 1$. For instance, a financial advisor who ignores the preferences asymmetry of her GDA client with $\kappa = 0.97$ and treats him as an EU investor with equal effective risk aversion recommends to invest 8.24% more in stock, 6.78% less in bond and 1.46% less in cash relative to the optimal allocation. The portfolio recommendation to the GDA client with $\kappa = 1.02$ would instead be 7.39% less in stock, 6.08% more in bond and 1.31% more in cash relative to the optimal allocation. Interestingly, Table 6 also shows that although it is still very small, the certainty equivalent cost of ignoring preferences asymmetry is about twice or more the cost of ignoring returns asymmetry for all base case GDA investors.

3.2.4 The Effect of Loss Aversion

While Tables 2 to 4 show results only for values of effective risk aversion corresponding to our base case value of loss aversion $\ell = 2$, we have also computed portfolio allocations for a continuum of values of loss aversion and they are shown in Figure 2; as in Campbell et al. (2003), the horizontal axis shows effective risk tolerance $1/\gamma$ rather than effective risk aversion γ, in order to display the behavior of highly conservative investors more compactly, and because portfolio demands are linear in risk tolerance as shown by equation (19). Infinitely conservative investors with $1/\gamma = 0$ are plotted at the right edge of the figure, so that as the eye moves from left to right we see the effects of increasing risk aversion on asset allocation. In each graph, effective risk aversion varies because loss aversion varies, everything else equal, so the graphs show the effect of loss aversion. Interestingly, these graphs also allow comparisons of different types of investors when they all have equal effective risk aversion.

Panel A1 of Figure 2 shows that risk tolerance decreases as loss aversion increases, and becomes zero for DA investors with $\kappa = 1$ when $\ell \geq \ell^*$. Thus, DA investors with $\ell \geq \ell^*$ will not hold risky securities, consistent with the results of Ang et al. (2005) regarding (non-) market participation of disappointment averse investors. To the contrary of DA investors, GDA investors will always participate to the stock market and, the farther the hurdle rate from the certainty equivalent the higher the risk tolerance. For example, risk tolerance is higher for our base case GDA investor with $\kappa = 0.96$ compared to the one with $\kappa = 0.98$; it is also higher for the GDA investor with $\kappa = 1.04$ relative to the one with $\kappa = 1.02$. Panel A2 shows that, at equal level of effective risk tolerance, implicit asymmetry aversion is positive for EU investors and for GDA investors with $\kappa < 1$, while it is negative for DA investors and for GDA investors with $\kappa > 1$, confirming our previous discussion from Table 2. As loss aversion increases, the willingness of GDA investors with $\kappa < 1$ to hedge against large downside losses and that of GDA investors with $\kappa > 1$ to bet on large upside gains also increases as reflected in the larger magnitudes of their implicit asymmetry aversion coefficient as shown in Panel A2.

Panels B1 and B2 of Figure 2 show that, at equal level of effective risk tolerance, stock demand decreases with κ while bond demand increases with κ, reflecting the manager's attitude from speculative to hedging as reflected in the decreasing hurdle rate as shown by Panel A3. In consequence, the bond/stock allocation ratio also decreases with κ at equal effective risk aversion as shown in Panel B3 of Figure 2, where effective risk aversion is proxied by the stock demand as in Canner et al. (1997). As loss aversion increases, all types of investors decrease both stock and bond demands; however, stock demand decreases proportionally more than bond demand for GDA investors with $\kappa < 1$ consistent with a larger positive asymmetry aversion coefficient as shown in Panel A2, increasing the bond/stock allocation ratio, while it is the contrary for DA investors and GDA investors with $\kappa > 1$ consistent with a larger negative asymmetry aversion coefficient, decreasing the bond/stock allocation ratio as shown in Panel B3 of Figure 2. The bond/stock allocation ratio of EU and DA investors does not vary much with effective risk aversion, and remains slightly above and slightly below the mean-variance level, respectively.

Panels A1 and A2 of Figure 3 display changes in stock and bond demands respectively, between the suboptimal portfolio strategy resulting from ignoring asymmetry in the distribution of log asset returns and the optimal strategy, while Panel A3 shows the certainty equivalent cost of ignoring returns asymmetry. As loss aversion increases, suboptimal portfolio shifts become more important, reflecting the absence of the asymmetry-variance fund and thus the inability of GDA investors who would like to do so to implement hedging (if $\kappa < 1$) or speculative (if $\kappa > 1$) strategies in order to limit large downside losses or chase large upside gains. In consequence, the certainty equivalent cost for this inability resulting from ignoring returns asymmetry also increases.

Similarly, Panels B1 and B2 of Figure 3 display changes in stock and bond demands respectively, between the suboptimal portfolio strategy resulting from ignoring preferences asymmetry, while Panel B3 shows the associated certainty equivalent cost. As we previously discussed, EU investors invest very little in the asymmetry-variance fund compared to GDA investors with equal effective risk aversion and even hold positions that are contrary to those of their GDA counterparts with $\kappa > 1$. Thus, treating GDA investors as EU investors with equal effective risk aversion results in portfolio recommendations that seriously undermine their hedging and speculative strategies, resulting in suboptimal shifts which become more important as loss aversion increases. In consequence, the certainty equivalent cost for wrong recommendations resulting from ignoring preferences asymmetry also increases. Interestingly, Figure 3 also shows that although they are very small, the certainty equivalent cost of ignoring preferences asymmetry is about twice or more the cost of ignoring returns asymmetry for all base case GDA investors.

3.2.5 The Effect of Time Horizon

Popular advisors tend to recommend that an investor's time horizon, as well as his tolerance toward risk, should influence the composition of his portfolio. According to these advisors, younger investors – who have long time horizons – should invest more aggressively than older investors. In this section we discuss the effect of time horizon on investor's attitude toward risk and asset allocation. While Tables 2 to 4 show results corresponding to our base case investment horizon $T = 1$ month, we have also computed portfolio allocations for a continuum of values of the investment horizon up to sixty months (or five years).

[To be completed.]

4 Conclusion and Future Work

[To be written.]

Apppendix

A Derivations

A.1 Certainty Equivalent and Optimal Allocation Policy

Recalling that $r_{F,T} = \ln R_{F,T}$ and $\eta = \ln(\mathcal{R})$, we have

$$\begin{aligned} U(\kappa\mathcal{R}) - U(R_{F,T}) &= U(\kappa\mathcal{R})\left(1 - \frac{U(R_{F,T})}{U(\kappa\mathcal{R})}\right) = U(\kappa\mathcal{R})\left(1 - \left(\frac{R_{F,T}}{\kappa\mathcal{R}}\right)^{-\alpha}\right) \\ &= U(\kappa\mathcal{R})\left(1 - \exp\left(-\alpha\left(r_{F,T} - \ln\kappa - \eta\right)\right)\right) \\ &= U(\kappa\mathcal{R})\left(1 - \exp\left(\alpha\left(\ln\kappa + \eta - r_{F,T}\right)\right)\right). \end{aligned} \tag{A.1}$$

Observing that $\forall a, X \in \mathbb{R}$ we have

$$\left(1 - \exp(aX)\right) I(X > 0) = 1 - \exp\left(aX I(X > 0)\right) = 1 - \exp\left(a \max(X, 0)\right), \tag{A.2}$$

we show that

$$\begin{aligned} &E\left[\left(U(\kappa\mathcal{R}) - U(R_{F,T})\right) I\left(R_{F,T} < \kappa\mathcal{R}\right)\right] \\ &= U(\kappa\mathcal{R}) E\left[\left(1 - \exp\left(\alpha\left(\ln\kappa + \eta - r_{F,T}\right)\right)\right) I\left(r_{F,T} < \ln\kappa + \eta\right)\right] \\ &= \kappa^{-\alpha} U(\mathcal{R}) E\left[\left(1 - \exp\left(\alpha\left(\ln\kappa + \eta - r_{F,T}\right)\right)\right) I\left(\ln\kappa + \eta - r_{F,T} > 0\right)\right] \\ &= \kappa^{-\alpha} U(\mathcal{R}) \left(1 - E\left[\exp\left(\alpha p_{F,T}\right)\right]\right), \end{aligned} \tag{A.3}$$

where

$$p_{F,T} \equiv \max\left(\ln\kappa + \eta - r_{F,T}, 0\right).$$

Substituting out equation (A.3) in equation (5) and solving for $U(\mathcal{R})$, we arrive at

$$U(\mathcal{R}) = \frac{E\left[U\left(R_{F,T}\right)\right]}{\theta + \ell\kappa^{-\alpha}\left(1 - E\left[\exp\left(\alpha p_{F,T}\right)\right]\right)}. \tag{A.4}$$

Simplifying both sides and taking logs, we get

$$\ln \mathcal{R}^{-\alpha} = \ln E\left[R_{F,T}^{-\alpha}\right] - \ln\left(\theta + \ell\kappa^{-\alpha}\left(1 - E\left[\exp\left(\alpha p_{F,T}\right)\right]\right)\right), \tag{A.5}$$

which finally leads to equation (6), which also defines an implicit function

$$G(w, \eta) = 0, \tag{A.6}$$

with

$$G(w, \eta) = -\eta - \frac{1}{\alpha}\ln E\left[\exp\left(-\alpha r_{F,T}\right)\right] + \frac{1}{\alpha}\ln\left(\theta + \ell\kappa^{-\alpha}\left(1 - E\left[\exp\left(\alpha p_{F,T}\right)\right]\right)\right). \tag{A.7}$$

If an optimal allocation policy does exist, then it satisfies the necessary condition

$$\frac{\partial \eta}{\partial w} = 0. \tag{A.8}$$

Implicit differentiation of equation (A.6) implies that

$$\frac{\partial \eta}{\partial w} = -\frac{G_1'(w, \eta)}{G_2'(w, \eta)}, \tag{A.9}$$

where G_1' is the partial derivative of G with respect to its first argument and G_2' is the partial derivative of G with respect to its second argument. Finally,

$$\frac{\partial \eta}{\partial w} = 0 \quad \text{is equivalent to} \quad G_1'(w,\eta) = 0. \tag{A.10}$$

Computing $G_1'(w,\eta)$ from (A.7), we have

$$\begin{aligned} G_1'(w,\eta) &= \frac{E\left[(\partial r_{F,T}/\partial w)\exp\left(-\alpha r_{F,T}\right)\right]}{E\left[\exp\left(-\alpha r_{F,T}\right)\right]} - \frac{\ell\kappa^{-\alpha}E\left[(\partial p_{F,T}/\partial w)\exp\left(\alpha p_{F,T}\right)\right]}{\theta+\ell\kappa^{-\alpha}\left(1-E\left[\exp\left(\alpha p_{F,T}\right)\right]\right)} \\ &= \frac{E\left[(\partial r_{F,T}/\partial w)\exp\left(-\alpha r_{F,T}\right)\right]}{E\left[\exp\left(-\alpha r_{F,T}\right)\right]} - c\frac{E\left[(\partial p_{F,T}/\partial w)\exp\left(\alpha p_{F,T}\right)\right]}{E\left[\exp\left(\alpha p_{F,T}\right)\right]} \\ &= E\left[H_T\frac{\partial r_{F,T}}{\partial w}\right] - cE\left[D_T\frac{\partial p_{F,T}}{\partial w}\right] \\ &= E^{\mathbf{H}}\left[\frac{\partial r_{F,T}}{\partial w}\right] - cE^{\mathbf{D}}\left[\frac{\partial p_{F,T}}{\partial w}\right], \end{aligned} \tag{A.11}$$

where $c > 0$ is given by equation (8) and where $E^{\mathbf{H}}[\cdot]$ and $E^{\mathbf{D}}[\cdot]$ respectively denote the expectation operators associated with the densities H_T and D_T, defined by

$$H_T = \frac{\exp\left(-\alpha r_{F,T}\right)}{E\left[\exp\left(-\alpha r_{F,T}\right)\right]} \quad \text{and} \quad D_T = \frac{\exp\left(\alpha p_{F,T}\right)}{E\left[\exp\left(\alpha p_{F,T}\right)\right]}. \tag{A.12}$$

We also have that

$$\frac{\partial r_{F,T}}{\partial w} = \left(r_T - r_f\iota + \frac{1}{2}\omega\right) - \Omega w \quad \text{and} \quad \frac{\partial p_{F,T}}{\partial w} = -\frac{\partial r_{F,T}}{\partial w} I\left(r_{F,T} < \ln\kappa + \eta\right), \tag{A.13}$$

which substituting out into the last form of (A.11) yields

$$
\begin{aligned}
G_1'(w,\eta) &= E^{\mathbf{H}}\left[\left(r_T - r_f\iota + \frac{1}{2}\omega\right)\right] + cE^{\mathbf{D}}\left[\left(r_T - r_f\iota + \frac{1}{2}\omega\right) I\left(r_{F,T} < \ln\kappa + \eta\right)\right] \\
&\quad - \left(1 + cE^{\mathbf{D}}\left[I\left(r_{F,T} < \ln\kappa + \eta\right)\right]\right)\Omega w \\
&= E^{\mathbf{H}}\left[\left(r_T - r_f\iota + \frac{1}{2}\omega\right)\right] + c\pi^{\mathbf{D}}E^{\mathbf{D}}\left[\left(r_T - r_f\iota + \frac{1}{2}\omega\right) \mid r_{F,T} < \ln\kappa + \eta\right] \\
&\quad - \left(1 + c\pi^{\mathbf{D}}\right)\Omega w,
\end{aligned}
\tag{A.14}
$$

then

$$
G_1'(w,\eta) = \left(m^{\mathbf{H}} - r_f\iota + \frac{1}{2}\omega\right) + c\pi^{\mathbf{D}}\left(m^{\mathbf{H}-} - r_f\iota + \frac{1}{2}\omega\right) - \left(1 + c\pi^{\mathbf{D}}\right)\Omega w, \tag{A.15}
$$

where

$$
\begin{aligned}
&m^{\mathbf{H}} \equiv E^{\mathbf{H}}\left[r_T\right] \quad \text{and} \quad m^{\mathbf{H}-} \equiv E^{\mathbf{H}}\left[r_T \mid r_{F,T} < \ln\kappa + \eta\right] = E^{\mathbf{D}}\left[r_T \mid r_{F,T} < \ln\kappa + \eta\right] \\
&\pi^{\mathbf{D}} \equiv Prob^{\mathbf{D}}\left(r_{F,T} < \ln\kappa + \eta\right) = E^{\mathbf{D}}\left[I\left(r_{F,T} < \ln\kappa + \eta\right)\right].
\end{aligned}
\tag{A.16}
$$

Finally, the necessary condition for an optimal allocation policy, $G_1'(w,\eta) = 0$, implies that

$$
w = \nu w^{\mathbf{H}} + (1-\nu)\, w^{\mathbf{H}-} \tag{A.17}
$$

where

$$
w^{\mathbf{H}} \equiv \Omega^{-1}\left(m^{\mathbf{H}} - r_f\iota + \frac{1}{2}\omega\right), \quad w^{\mathbf{H}-} \equiv \Omega^{-1}\left(m^{\mathbf{H}-} - r_f\iota + \frac{1}{2}\omega\right) \quad \text{and} \quad \nu \equiv \frac{1}{1 + c\pi^{\mathbf{D}}}. \tag{A.18}
$$

A.2 Multiperiod Asset Returns

Let $r_t(T)$ denotes the vector of log returns over the period from $t-T$ to t. We have

$$\begin{aligned} r_t(T) &\equiv \sum_{j=1}^{T} r_{t-j+1} = \phi_0 T + \phi_1 z_{0,t}(T) + z_t(T) \\ \text{where} \quad z_{0,t}(T) &\equiv \sum_{j=1}^{T} \varepsilon_{0,t-j+1} \quad \text{and} \quad z_t(T) \equiv \sum_{j=1}^{T} z_{t-j+1}. \end{aligned} \tag{A.19}$$

The aggregate process $z_{0,t}(T)$ has a nontrivial distribution. Its cumulant-generating function is equal to T times the cumulant-generating function of $z_{0,t}$. We exploit this property to approximate the distribution of $z_{0,t}(T)$ with a truncated normal distribution as follows:

$$z_{0,t}(T) \approx d(T) + \nu(T)\,\varepsilon_{0,t}(T) \tag{A.20}$$

where $\nu(T) > 0$ and $\varepsilon_{0,t}(T)$ has a standard normal distribution truncated from below at $-\tau(T)$. We then solve for the values of $d(T)$, $\nu(T)$ and $\tau(T)$ so that the mean, the variance and the nth cumulant are matched, with $n = 3$ or $n = 4$. Formally, $d(T)$, $\nu(T)$ and $\tau(T)$ are solutions to the system

$$\begin{cases} d(T) + \nu(T)\,\xi_1(\tau(T)) = T\xi_1(\tau) \\ \nu^2(T)\,(1 + \xi_2(\tau(T))) = T\,(1 + \xi_2(\tau)) \\ \nu^n(T)\,\xi_n(\tau(T)) = T\xi_n(\tau) \end{cases} \tag{A.21}$$

where

$$\xi_1(x) = \frac{\phi(x)}{\Phi(x)} \quad \text{and} \quad \xi_2(x) = -\xi_1(x)(x+\xi_1(x))$$
$$\xi_3(x) = -\xi_2(x)(x+\xi_1(x)) - \xi_1(x)(1+\xi_2(x)) \qquad \text{(A.22)}$$
$$\xi_4(x) = -\xi_3(x)(x+2\xi_1(x)) - 2\xi_2(x)(1+\xi_2(x))$$
$$\xi_5(x) = -\xi_4(x)(x+2\xi_1(x)) - 3\xi_3(x)(1+2\xi_2(x)),$$

and where $\phi(x)$ and $\Phi(x)$ are respectively the standard normal probability and cumulative distribution functions. More precisely, the functions $\xi_j(x)$, $j = 1, 2, \ldots$ are the jth order derivatives of the function $\xi_0(x) = \ln \Phi(x)$ and can be computed recursively, such as shown in equation (A.22) for the first five derivatives.

Given the homoscedastic vector autoregression formulation in equation (12), the unconditional distribution of the k-dimensional vector process z_t can be derived as a multivariate gaussian distribution with mean zero and autocovariance matrices $\Gamma_z(j)$, $j = 0, 1, 2, \ldots$ given by

$$\Gamma_z(j) = E\left[z_t z_{t-j}^{\top}\right] = \Phi_z^j \Omega_z \quad \text{where} \quad vec(\Omega_z) = \left[Id_{k^2} - (\Phi_z \otimes \Phi_z)\right]^{-1} vec(\Omega_u), \qquad \text{(A.23)}$$

and where Id_k denotes the $k \times k$ identity matrix. It follows that the aggregate process $z_t(T)$ is a k-dimensional vector gaussian process with mean zero and covariance matrix given by

$$\Omega_z(T) = T\Gamma_z(0) + \sum_{j=1}^{T-1} (T-j)\left(\Gamma_z(j) + \Gamma_z^{\top}(j)\right). \qquad \text{(A.24)}$$

Let $\omega_{z,i}(T)$, $i = 1, 2, \ldots, k$ denote the diagonal elements of the matrix $\Omega_z(T)$, and let $\Psi(T)$ denote the correlation matrix associated with the covariance matrix $\Omega_z(T)$.

It follows that T-period log returns on assets may be written

$$r_{i,t}(T) = \mu_i(T) + \sigma_i(T)\left[\delta_i(T)\,\varepsilon_{0,t}(T) + \sqrt{1-\delta_i^2(T)}\varepsilon_{i,t}(T)\right], \quad i = 1, 2, \ldots, k \qquad \text{(A.25)}$$

where $\varepsilon_{0,t}(T)$ is the common shock with standard normal distribution truncated from below at $-\tau(T)$, and the vector of assets' specific shocks $\varepsilon_t(T) = (\varepsilon_{1,t}(T), \varepsilon_{2,t}(T), \ldots, \varepsilon_{k,t}(T))^\top$ is k-dimensional normal with standardized marginals, independent of $\varepsilon_{0,t}(T)$, with the correlation matrix $\Psi(T)$, and where the horizon-dependent coefficients in (13) are given by

$$\begin{aligned}&\mu_i(T) = \phi_{0,i}T + \phi_{1,i}d(T), \quad \sigma_i(T) = \sqrt{\phi_{1,i}^2\nu^2(T) + \omega_{z,i}(T)}\\ &\text{and} \quad \delta_i(T) = \frac{\phi_{1,i}\nu(T)}{\sqrt{\phi_{1,i}^2\nu^2(T) + \omega_{z,i}(T)}}.\end{aligned} \qquad \text{(A.26)}$$

In the paper, we focus on the period from date 0 to date T without loss of generality, setting $t = T$ in equation (A.25). For ease of readability, we also use short-hand notations $r_{i,T}$, μ_i, σ_i, δ_i, Ψ and $\varepsilon_{i,T}$ for $r_{i,T}(T)$, $\mu_i(T)$, $\sigma_i(T)$, $\delta_i(T)$, $\Psi(T)$ and $\varepsilon_{i,T}(T)$ respectively, leading to equation (13) in the paper.

A.3 Solution under Skew-Normal Asset Log Returns

We show that for $u \in \mathbb{R}$ and $v \in \mathbb{R}^k$, we have

$$ur_{F,T} + v^\top r_T = \bar{\mu}(u,v) + \bar{\sigma}(u,v)\left[\bar{\delta}(u,v)\,\varepsilon_0 + \sqrt{1-\bar{\delta}^2(u,v)}\bar{\varepsilon}_T(u,v)\right] \qquad \text{(A.27)}$$

where

$$\bar{\mu}(u,v) = u\left[w^\top\left(\mu - \iota r_f + \frac{1}{2}\omega\right) - \frac{1}{2}w^\top\Omega w + r_f\right] + v^\top\mu$$
$$\bar{\sigma}^2(u,v) = (uw+v)^\top\left((\Lambda\Psi\Lambda) + aa^\top\right)(uw+v) \tag{A.28}$$
$$\bar{\delta}(u,v) = \frac{(uw+v)^\top a}{\bar{\sigma}(u,v)} \quad \text{and} \quad \bar{\varepsilon}_T(u,v) = \frac{(uw+v)^\top \Lambda\varepsilon_T}{\sqrt{(uw+v)^\top(\Lambda\Psi\Lambda)(uw+v)}},$$

where $\lambda_i = \sigma_i\sqrt{1-\delta_i^2}$ for $i = 1, 2, \ldots, k$ and $\Lambda = Diag(\lambda_1, \lambda_2, \ldots, \lambda_k)$ is a diagonal matrix with specified diagonal elements. It follows that, similar to individual assets, the fund log return may be written as in equation (17) with parameters defined in equation (18).

Let $\bar{\psi}(u,v)$ denote the correlation between the variables $\varepsilon_{F,T}$ and $\bar{\varepsilon}_T(u,v)$. We have

$$\bar{\psi}(u,v) \equiv Cov\left(\varepsilon_{F,T}, \bar{\varepsilon}_T(u,v)\right) = \frac{(uw+v)^\top(\Lambda\Psi\Lambda)w}{\sqrt{\left[(uw+v)^\top(\Lambda\Psi\Lambda)(uw+v)\right]\left[w^\top(\Lambda\Psi\Lambda)w\right]}} \tag{A.29}$$

Also let $\bar{\rho}(u,v)$ denote the following correlation coefficient

$$\begin{aligned}\bar{\rho}(u,v) &= \bar{\delta}(u,v)\,\delta_F + \bar{\psi}(u,v)\sqrt{1-\bar{\delta}^2(u,v)}\sqrt{1-\delta_F^2} \\ &= \bar{\delta}(u,v)\,\delta_F + \frac{(uw+v)^\top(\Lambda\Psi\Lambda)w}{\bar{\sigma}(u,v)\,\sigma_F}\end{aligned} \tag{A.30}$$

In order to compute expectations under the manager's regular risk-adjusted density, H_T, and downside risk-adjusted density, D_T, we make use of the following lemma.

Lemma A.1 *Let*

$$r_1 = \mu_1 + \sigma_1\left[\delta_1\varepsilon_0 + \sqrt{1-\delta_1^2}\varepsilon_1\right] \quad \text{and} \quad r_2 = \mu_2 + \sigma_2\left[\delta_2\varepsilon_0 + \sqrt{1-\delta_2^2}\varepsilon_2\right]$$

where the variable ε_0 has a standard normal distribution truncated from below at $-\tau$, and ε_1 and ε_2 are two standard normal variables independent of ε_0 with correlation coefficient ψ, and let's consider the correlation coefficient $\rho = \delta_1\delta_2 + \psi\sqrt{1-\delta_1^2}\sqrt{1-\delta_2^2}$. Then, we have

$$
\begin{aligned}
&E\left[\exp\left(vr_2\right) I\left(r_1 < x\right)\right] \\
&\quad = \exp\left(\mu_2 v + \frac{1}{2}\sigma_2^2 v^2\right) \Phi_2\left(\frac{x-\mu_1}{\sigma_1} - \left(\sigma_2\rho\right)v, \tau + \left(\sigma_2\delta_2\right)v; -\delta_1\right) / \Phi\left(\tau\right)
\end{aligned}
$$

where $\phi(\cdot)$ denotes the standard normal probability distribution function, $\Phi(\cdot)$ denotes the standard normal cumulative distribution function, and $\Phi_2(\cdot, \cdot; c)$ denotes the cumulative distribution function of a bivariate standard normal with correlation coefficient c.

The proof of Lemma A.1 is trivial but necessitates a few algebraic steps. It exploits the independence between the bivariate normal and the truncated normal in expressing the bivariate extended skew-normal, and makes use of the law of iterated expectation by first conditioning on the truncated normal.

We now consider the function $M(u, v; x)$ defined by

$$
M(u, v; x) \equiv E\left[\exp\left(ur_{F,T} + v^\top r_T\right) I\left(r_{F,T} < x\right)\right]. \tag{A.31}
$$

By applying Lemma A.1, we show that

$$
M(u, v; x) = \exp\left(\bar{\mu}(u, v) + \frac{1}{2}\bar{\sigma}^2(u, v)\right) \frac{\Phi_2\left(p(u, v; x), q(u, v); -\delta_F\right)}{\Phi(\tau)} \tag{A.32}
$$

where

$$
p(u, v; x) = \frac{x - \mu_F}{\sigma_F} - \bar{\sigma}(u, v)\,\bar{\rho}(u, v) \quad \text{and} \quad q(u, v) = \tau + \bar{\sigma}(u, v)\,\bar{\delta}(u, v). \tag{A.33}
$$

The partial derivative of $M(u,v;x)$ with respect to $y \in \{u,v\}$ can easily be computed from the *Leibniz integral rule* and the *fundamental theorem of calculus*. Let

$$M'_y(u,v;x) \equiv \frac{\partial M(u,v;x)}{\partial y}. \tag{A.34}$$

It follows that

$$\begin{aligned} M'_y(u,v;x) = \frac{1}{\Phi(\tau)} \exp\left(\bar{\mu}(u,v) + \frac{1}{2}\bar{\sigma}^2(u,v)\right) \times \\ \left[\left(\frac{\partial \bar{\mu}(u,v)}{\partial y} + \frac{1}{2}\frac{\partial \bar{\sigma}^2(u,v)}{\partial y}\right)\Phi_2\left(p(u,v;x), q(u,v); -\delta_F\right)\right. \\ + \frac{\partial p(u,v;x)}{\partial y}\phi\left(p(u,v;x)\right)\Phi\left(\frac{\delta_F p(u,v;x) + q(u,v)}{\sqrt{1-\delta_F^2}}\right) \\ \left. + \frac{\partial q(u,v)}{\partial y}\phi\left(q(u,v)\right)\Phi\left(\frac{p(u,v;x) + \delta_F q(u,v)}{\sqrt{1-\delta_F^2}}\right)\right]. \end{aligned} \tag{A.35}$$

We have

$$\begin{aligned} p(u,v;x) &= \frac{x-\mu_F}{\sigma_F} - \frac{(uw+v)^\top \left((\Lambda\Psi\Lambda) + aa^\top\right) w}{\sigma_F} \\ q(u,v) &= \tau + (uw+v)^\top a \end{aligned} \tag{A.36}$$

and also

$$\begin{aligned} &\frac{\partial \bar{\mu}(u,v)}{\partial u} = \mu_F \quad \text{and} \quad \frac{\partial \bar{\mu}(u,v)}{\partial v} = \mu \quad \text{and} \quad \frac{\partial q(u,v)}{\partial u} = a_F \quad \text{and} \quad \frac{\partial q(u,v)}{\partial v} = a \\ &\frac{\partial \bar{\sigma}^2(u,v)}{\partial u} = 2u\sigma_F^2 + 2v^\top \left((\Lambda\Psi\Lambda) + aa^\top\right) w \\ &\frac{\partial \bar{\sigma}^2(u,v)}{\partial v} = 2\left((\Lambda\Psi\Lambda) + aa^\top\right)(uw+v) \\ &\frac{\partial p(u,v;x)}{\partial u} = -\sigma_F \quad \text{and} \quad \frac{\partial p(u,v;x)}{\partial v} = -\frac{1}{\sigma_F}\left((\Lambda\Psi\Lambda) + aa^\top\right) w. \end{aligned} \tag{A.37}$$

In the following, we use the fact that:

$$m = \mu + a\xi_1(\tau) \quad \text{and} \quad \Omega = \left((\Lambda\Psi\Lambda) + aa^\top\right) + \xi_2(\tau)\, aa^\top. \tag{A.38}$$

Observe that we have

$$\begin{aligned} m^{\mathbf{H}} &= \frac{M'_v(-\alpha, 0; +\infty)}{M(-\alpha, 0; +\infty)} \\ &= \mu - \alpha\left((\Lambda\Psi\Lambda) + aa^\top\right) w + a\tau_0 \\ &= m - \alpha\left(\Omega w - \xi_2(\tau)\, aa_F\right) + a\left(\tau_0 - \xi_1(\tau)\right), \end{aligned} \tag{A.39}$$

where

$$\tau_0 = \frac{\phi(\tau - \alpha a_F)}{\Phi(\tau - \alpha a_F)} \tag{A.40}$$

and where we have used the facts that $p(-\alpha, 0; +\infty) = +\infty$ and $q(-\alpha, 0) = \tau - \alpha a_F$.

Also observe that the downside risk-adjusted and the physical disappointment probabilities are given by

$$\begin{aligned} \pi^{\mathbf{P}} &\equiv Prob\left(r_{F,T} < \ln\kappa + \eta\right) \\ &= E\left[I\left(r_{F,T} < \ln\kappa + \eta\right)\right] \\ &= M\left(0, 0; \ln\kappa + \eta\right) \\ &= \Phi_2\left(d_1, \tau; -\delta_F\right)/\Phi(\tau) \\ \pi^{\mathbf{D}} &= \frac{\exp\left(\alpha\left(\ln\kappa + \eta\right)\right) M\left(-\alpha, 0; \ln\kappa + \eta\right)}{\exp\left(\alpha\left(\ln\kappa + \eta\right)\right) M\left(-\alpha, 0; \ln\kappa + \eta\right) + \left(1 - M\left(0, 0; \ln\kappa + \eta\right)\right)} \\ &= \frac{A\Phi_2\left(d_2, \tau - \alpha a_F; -\delta_F\right)/\Phi(\tau)}{1 + \left(A\Phi_2\left(d_2, \tau - \alpha a_F; -\delta_F\right) - \Phi_2\left(d_1, \tau; -\delta_F\right)\right)/\Phi(\tau)} \end{aligned} \tag{A.41}$$

where

$$A \equiv \exp\left(\alpha\left(\ln\kappa+\eta-\mu_F\right)+\frac{1}{2}\alpha^2\sigma_F^2\right)=\exp\left(\alpha\sigma_F\left(d_1+\frac{1}{2}\alpha\sigma_F\right)\right),$$
$$d_1 \equiv p\left(0,0;\ln\kappa+\eta\right)=\frac{\ln\kappa+\eta-\mu_F}{\sigma_F} \quad\text{and}\quad d_2\equiv p\left(-\alpha,0;\ln\kappa+\eta\right)=d_1+\alpha\sigma_F, \tag{A.42}$$

and where η is the solution to the equation

$$\eta=\mu_F-\frac{1}{2}\alpha\sigma_F^2+\frac{1}{\alpha}\ln\left(\frac{\theta-\ell\kappa^{-\alpha}\left(A\Phi_2\left(d_2,\tau-\alpha a_F;-\delta_F\right)-\Phi_2\left(d_1,\tau;-\delta_F\right)\right)/\Phi\left(\tau\right)}{\Phi\left(\tau-\alpha a_F\right)/\Phi\left(\tau\right)}\right). \tag{A.43}$$

Next, observe that

$$\begin{aligned} m^{\mathbf{H}-} &= \frac{M_v'\left(-\alpha,0;\ln\kappa+\eta\right)}{M\left(-\alpha,0;\ln\kappa+\eta\right)} \\ &= \mu-\left(\alpha+\frac{\tau_1}{\sigma_F}\right)\left(\left(\Lambda\Psi\Lambda\right)+aa^\top\right)w+a\tau_2 \\ &= m-\left(\alpha+\frac{\tau_1}{\sigma_F}\right)\left(\Omega w-\xi_2\left(\tau\right)aa_F\right)+a\left(\tau_2-\xi_1\left(\tau\right)\right), \end{aligned} \tag{A.44}$$

where

$$\begin{aligned} \tau_1 &= \frac{\phi\left(d_2\right)\Phi\left(\left(1/\sqrt{1-\delta_F^2}\right)\left(\delta_F d_1+\tau\right)\right)}{\Phi_2\left(d_2,\tau-\alpha a_F;-\delta_F\right)} \\ \tau_2 &= \frac{\phi\left(\tau-\alpha a_F\right)\Phi\left(\left(1/\sqrt{1-\delta_F^2}\right)\left(d_1+\delta_F\tau\right)+\alpha\sigma_F\sqrt{1-\delta_F^2}\right)}{\Phi_2\left(d_2,\tau-\alpha a_F;-\delta_F\right)}. \end{aligned} \tag{A.45}$$

Substituting out equations (A.39) and (A.44) into equation (A.18), then (A.17), and solving for the optimal allocation policy w, after some algebra we arrive at equation (19),

where

$$\gamma = 1 + \nu\alpha + (1-\nu)\left(\alpha + \frac{\tau_1}{\sigma_F}\right) = 1 + \alpha + (1-\nu)\frac{\tau_1}{\sigma_F}$$
$$\chi = \nu\left[(\tau_0 - \xi_1(\tau)) + \xi_2(\tau)\alpha a_F\right] + (1-\nu)\left[(\tau_2 - \xi_1(\tau)) + \xi_2(\tau)\left(\alpha + \frac{\tau_1}{\sigma_F}\right)a_F\right]. \qquad \text{(A.46)}$$

The sensitivity of the manager's objective to the put option payoff, defined in equation (8) obtains as

$$c = \frac{\ell\kappa^{-\alpha}\left(1 + \left(A\Phi_2\left(d_2, \tau - \alpha a_F; -\delta_F\right) - \Phi_2\left(d_1, \tau; -\delta_F\right)\right)/\Phi(\tau)\right)}{\theta - \ell\kappa^{-\alpha}\left(A\Phi_2\left(d_2, \tau - \alpha a_F; -\delta_F\right) - \Phi_2\left(d_1, \tau; -\delta_F\right)\right)/\Phi(\tau)}. \qquad \text{(A.47)}$$

Finally, the loss probability, the fund expected shortfall and the vector of asset marginal expected shortfalls are respectively defined by

$$\pi^{\mathbf{L}} \equiv Prob\left(r_{F,T} < x\right) = E\left[I\left(r_{F,T} < x\right)\right] = M(0,0;x)$$
$$m_F^- - r_f + \frac{1}{2}\omega_F \equiv E\left[\left(r_{F,T} - r_f + \frac{1}{2}\omega_F\right) \mid r_{F,T} < x\right] = \frac{M_u'(0,0;x)}{M(0,0;x)} - r_f + \frac{1}{2}\omega_F \qquad \text{(A.48)}$$
$$m^- - r_f\iota + \frac{1}{2}\omega \equiv E\left[\left(r_T - r_f\iota + \frac{1}{2}\omega\right) \mid r_{F,T} < x\right] = \frac{M_v'(0,0;x)}{M(0,0;x)} - r_f\iota + \frac{1}{2}\omega$$

and given by

$$\pi^{\mathbf{L}} = \Phi_2\left(d_0, \tau; -\delta_F\right)/\Phi(\tau)$$
$$m_F^- = \mu_F - \tau_3\sigma_F + \tau_4 a_F \qquad \text{(A.49)}$$
$$m^- = \mu - \frac{\tau_3}{\sigma_F}\left(\Omega w - \vartheta_L - \xi_2(\tau)\, a a_F\right) + \tau_4 a$$

where

$$\tau_3 = \frac{\phi\left(d_0\right)\Phi\left(\left(\delta_F d_0 + \tau\right)/\sqrt{1-\delta_F^2}\right)}{\Phi_2\left(d_0, \tau; -\delta_F\right)} \quad \text{and} \quad \tau_4 = \frac{\phi\left(\tau\right)\Phi\left(\left(d_0 + \delta_F \tau\right)/\sqrt{1-\delta_F^2}\right)}{\Phi_2\left(d_0, \tau; -\delta_F\right)} \tag{A.50}$$

and

$$d_0 \equiv p\left(0, 0; x\right) = \frac{x - \mu_F}{\sigma_F}. \tag{A.51}$$

The corresponding upside potentials are also given by

$$\begin{aligned} m_F^+ - r_f + \frac{1}{2}\omega_F &\equiv E\left[\left(r_{F,T} - r_f + \frac{1}{2}\omega_F\right) \mid r_{F,T} \geq x\right] \\ m^+ - r_f \iota + \frac{1}{2}\omega &\equiv E\left[\left(r_T - r_f \iota + \frac{1}{2}\omega\right) \mid r_{F,T} \geq x\right] \end{aligned} \tag{A.52}$$

where

$$m_F^+ = \frac{m_F - \pi^{\mathbf{L}} m_F^-}{1 - \pi^{\mathbf{L}}} \quad \text{and} \quad m^+ = \frac{m - \pi^{\mathbf{L}} m^-}{1 - \pi^{\mathbf{L}}}. \tag{A.53}$$

References

Adcock, C. (2010). Asset pricing and portfolio selection based on the multivariate extended skew-student-t distribution, *Annals of Operations Research* **176**(1): 221–234.

Adcock, C. J. and Shutes, K. (2012). On the multivariate extended skew-normal, normal-exponential, and normal-gamma distributions, *Journal of Statistical Theory and Practice* **6**(4): 636–664.

Adcock, C. and Shutes, K. (2001). Portfolio selection based on the multivariate-skew normal distribution, *in* A. Skulimowski (ed.), *Financial Modelling*, Krakow: Progress & Business Publishers.

Allais, M. (1979). The foundations of a positive theory of choice involving risk and a criticism of the postulates and axioms of the american school (1952), *in* M. Allais and O. Hagen (eds), *Expected Utility Hypotheses and the Allais Paradox*, Vol. 21 of *Theory and Decision Library*, Springer Netherlands, pp. 27–145.

Ang, A. and Bekaert, G. (2002). International asset allocation with regime shifts, *Review of Financial Studies* **15**(4): 1137–1187.

Ang, A., Bekaert, G. and Liu, J. (2005). Why stocks may disappoint, *Journal of Financial Economics* **76**(3): 471 – 508.

Ang, A. and Chen, J. (2002). Asymmetric correlations of equity portfolios, *Journal of Financial Economics* **63**(3): 443–494.

Asparouhova, E., Bossaerts, P., Copic, J., Cornell, B., Cvitanic, J. and Meloso, D. (2012).

Competition in portfolio management: Theory and experiment, *Working Papers 438*, IGIER (Innocenzo Gasparini Institute for Economic Research), Bocconi University.

Azzalini, A. (1985). A class of distributions which includes the normal ones, *Scandinavian Journal of Statistics* **12**(2): 171–178.

Azzalini, A. (1986). Further results on class of distributions which includes the normal ones, *Statistica* **46**(2): 199–208.

Azzalini, A. (2005). The skew-normal distribution and related multivariate families*, *Scandinavian Journal of Statistics* **32**(2): 159–188.

Azzalini, A. and Dalla Valle, A. (1996). The multivariate skew-normal distribution, *Biometrika* **83**(4): 715–726.

Bajeux-Besnainou, I., Jordan, J. V. and Portait, R. (2001). An asset allocation puzzle: Comment, *American Economic Review* **91**(4): 1170–1179.

Barberis, N. (2000). Investing for the long run when returns are predictable, *Journal of Finance* **55**(1): 225–264.

Basak, S. and Makarov, D. (2013). Competition among portfolio managers and asset specialization, *Working Papers w0194*, Center for Economic and Financial Research (CEFIR).

Bernardo, A. E. and Ledoit, O. (2000). Gain, loss, and asset pricing, *Journal of Political Economy* **108**(1): 144–172.

Bonomo, M., Garcia, R., Meddahi, N. and Tédongap, R. (2011). Generalized disappointment aversion, long-run volatility risk and aggregate asset prices, *The Review of Financial Studies* **24**(1): 82–122.

Campbell, J. Y., Chan, Y. L. and Viceira, L. M. (2003). A multivariate model of strategic asset allocation, *Journal of Financial Economics* **67**(1): 41 – 80.

Campbell, J. Y. and Viceira, L. M. (2001). Who should buy long-term bonds?, *American Economic Review* **91**(1): 99–127.

Campbell, J. Y. and Viceira, L. M. (2002). Appendix for "strategic asset allocation: Portfolio choice for long-term investors".

Canner, N., Mankiw, N. G. and Weil, D. N. (1997). An asset allocation puzzle, *American Economic Review* **87**(1): 181–91.

Choi, S., Fisman, R., Gale, D. and Kariv, S. (2007). Consistency and heterogeneity of individual behavior under uncertainty, *American Economic Review* **97**(5): 1921–1938.

Das, S. R. and Uppal, R. (2004). Systemic risk and international portfolio choice, *The Journal of Finance* **59**(6): 2809–2834.

Epstein, L. G. and Zin, S. E. (1989). Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* **57**(4): 937–969.

Gill, D. and Prowse, V. L. (2012). A structural analysis of disappointment aversion in a real effort competition, *American Economic Review* **102**(1): 469–503.

Guidolin, M. and Timmermann, A. (2008). International asset allocation under regime switching, skew, and kurtosis preferences, *Review of Financial Studies* **21**(2): 889–935.

Gul, F. (1991). A theory of disappointment aversion, *Econometrica* **59**(3): 667–86.

Gupta, A. K., González-Farías, G. and Domínguez-Molina, J. A. (2004). A multivariate skew normal distribution, *Journal of Multivariate Analysis* **89**(1): 181 – 190.

Harvey, C. R., Liechty, J. C., Liechty, M. W. and Müller, P. (2010). Portfolio selection with higher moments, *Quantitative Finance* **10**(5): 469–485.

Hong, Y., Tu, J. and Zhou, G. (2006). Asymmetries in stock returns: Statistical tests and economic evaluation, *Review of Financial Studies* **20**(5): 1547–1581.

Jondeau, E. and Rockinger, M. (2006). Optimal portfolio allocation under higher moments, *European Financial Management* **12**(1): 29–55.

Kahneman, D. and Tversky, A. (1979). Prospect theory: An analysis of decision under risk, *Econometrica* **47**(2): 263–91.

Lintner, J. (1965). The valuation of risk assets and selection of risky investments in stock portfolios and capital budgets, *The Review of Economics and Statistics* **47**: 13–37.

Martellini, L. and Ziemann, V. (2010). Improved estimates of higher-order comoments and implications for portfolio selection, *Review of Financial Studies* **23**(4): 1467–1502.

Mossin, J. (1966). Equilibrium in a capital asset market, *Econometrica* **34**(4): 768–83.

Quiggin, J. (1982). A theory of anticipated utility, *Journal of Economic Behavior & Organization* **3**(4): 323–343.

Routledge, B. R. and Zin, S. E. (2010). Generalized disappointment aversion and asset prices, *Journal of Finance* **65**(4): 1303–1332.

Shalit, H. and Yitzhaki, S. (2003). An asset allocation puzzle: Comment, *American Economic Review* **93**(3): 1002–1008.

Sharpe, W. (1964). Capital asset prices: A theory of market equilibrium under conditions of risk, *The Journal of Finance* **19**: 425–442.

Simaan, Y. (1993). Portfolio selection and asset pricing—three-parameter framework, *Management Science* **39**(5): 568–577.

Vernic, R. (2006). Multivariate skew-normal distributions with applications in insurance, *Insurance: Mathematics and Economics* **38**(2): 413–426.

Table 1: Parameter estimates

The table presents parameter and moment estimates for our calibration of the model described in (13). The data used for the calibration is monthly log returns on three assets: the 30-day Treasury Bill (f), the 30-year government bond index (B), and the value-weighted index of the CRSP stocks (S). The period used is from January 1963 to December 2012. The table displays three different GMM estimation results maintaining the assumption that the returns are independently distributed across time ($\Phi_z = 0$ in (12)). GMM A is identified and fits the two means (m), the two volatilities ($\sqrt{\omega}$), the correlation ($corr$) and the two skewness (s). GMM B is overidentified, fitting the two coskewness ($coskew$) in addition to the seven moments considered in GMM A. Finally, GMM C is fitting the same moments as GMM A, but the skewness of bond is replaced by the coskewness of stock relative to bond. The top panel of the table shows sample moments and fitted moments. The values denoted by superscript ^{a} are not estimated, but are implied by the fitted distribution. The bottom panel shows parameter estimates in terms of the model described in (13). One-period returns ($T = 1$) were used for the estimation.

	sample	GMM A		GMM B		GMM C	
		estimate	s.e.	estimate	s.e.	estimate	s.e.
r_f	0.0038						
$m_S - r_f$	0.0046	0.0046	(0.0017)	0.0046	(0.0017)	0.0046	(0.0017)
$m_B - r_f$	0.0013	0.0013	(0.0008)	0.0013	(0.0008)	0.0013	(0.0008)
$\sqrt{\omega_S}$	0.0426	0.0426	(0.0022)	0.0426	(0.0022)	0.0426	(0.0022)
$\sqrt{\omega_B}$	0.0212	0.0212	(0.0012)	0.0212	(0.0012)	0.0212	(0.0012)
$corr_{SB}$	0.1046	0.1046	(0.0633)	0.1046	(0.0633)	0.1046	(0.0633)
s_S	-0.6448	-0.6448	(0.3540)	-0.6295	(0.3463)	-0.6448	(0.3540)
s_B	0.2019	0.2019	(0.2287)	0.0420	(0.0660)	0.0210^a	
$coskew_{SB}$	0.2059	0.4379^a		0.2552	(0.1291)	0.2059	(0.1416)
$coskew_{BS}$	-0.0670	-0.2973^a		-0.1035	(0.0365)	-0.0658^a	
κ_S	2.3962	1.1347^a		1.0989^a		1.1347^a	
κ_B	1.4714	0.2413^a		0.0297^a		0.0118^a	
μ_S		0.0364	(0.0064)	0.0361	(0.0063)	0.0364	(0.0064)
μ_B		-0.0095	(0.0037)	-0.0051	(0.0015)	-0.0038	(0.0029)
σ_S		0.1440	(0.0298)	0.1429	(0.0296)	0.1440	(0.0298)
σ_B		0.0512	(0.0169)	0.0348	(0.0069)	0.0305	(0.0098)
ψ		0.7364	(0.4030)	0.4557	(0.1792)	0.3924	(0.1884)
δ_S		-0.9783	(0.0157)	-0.9776	(0.0159)	-0.9783	(0.0157)
δ_B		0.9318	(0.0606)	0.8114	(0.0876)	0.7350	(0.2168)

Table 2: Portfolio Choice Implications: Investors' Attitude Towards Risk

The entries of the table are the effective risk aversion (γ), the implicit asymmetry aversion (χ), the log certainty equivalent (η) together with its expected utility component (η_1) and the downside risk penalty (η_2), the hurdle rate ($\ln \kappa + \eta$), the disappointment probability ($\pi^{\mathbf{P}}$), the expected shortfall (es_F) of the optimal portfolio and the associated marginal expected shortfalls of the stock (es_S) and the bond (es_B), the upside potential (up_F) of the optimal portfolio and the associated marginal upside potentials of the stock (up_S) and the bond (up_B). Excess risk aversion is fixed to $\alpha = 1$, and loss aversion to $\ell = 2$. The investment horizon is $T = 1$ month. The constant risk-free rate as well as log returns on stock and bond dynamics are calibrated with values reported in the panel GMM C of Table 1. Log certainty equivalent, expected shortfall and upside potential are in monthly percentage values.

	GDA Investor									EU Investor								
κ	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04
Effective Risk Aversion and Implicit Asymmetry Aversion																		
γ	6.28	8.15	11.90	23.15	∞	20.43	10.66	7.39	5.76	6.28	8.15	11.90	23.15	∞	20.43	10.66	7.39	5.76
χ	352	625	1396	5515		-1015	-260	-117	-66	15	27	63	260		200	49	22	12
c	2.08	2.06	2.04	2.02		2.00	2.00	2.00	2.00	0.00	0.00	0.00	0.00		0.00	0.00	0.00	0.00
Certainty Equivalent and Disappointment Threshold (Hurdle Rate)																		
η	0.58	0.53	0.49	0.44	0.38	0.46	0.53	0.59	0.64	0.54	0.50	0.46	0.42	0.38	0.43	0.47	0.51	0.55
η_1	0.61	0.56	0.51	0.45	0.38	0.49	0.57	0.64	0.69	0.54	0.50	0.46	0.42	0.38	0.43	0.47	0.51	0.55
η_2	-0.03	-0.03	-0.02	-0.01	0.00	-0.02	-0.04	-0.05	-0.05	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\ln \kappa + \eta$	-3.51	-2.51	-1.53	-0.57	0.38	1.46	2.51	3.55	4.56	-3.55	-2.55	-1.56	-0.58	0.38	1.42	2.45	3.47	4.47
Expected Shortfall and Upside Potential, Downside is Disappointing Event																		
$\pi^{\mathbf{P}}$	1.93	2.18	2.45	2.76		89.17	90.59	91.83	92.92	3.47	3.82	4.20	4.58		92.71	93.54	94.26	94.91
es_F	-4.71	-3.53	-2.35	-1.18		-0.04	-0.05	-0.03	0.01	-5.05	-3.80	-2.54	-1.28		0.00	0.02	0.05	0.09
es_S	-10.20	-9.86	-9.53	-9.22		-0.21	-0.12	-0.04	0.03	-10.01	-9.74	-9.49	-9.24		0.04	0.09	0.14	0.18
es_B	-2.27	-2.30	-2.32	-2.34		-0.07	-0.05	-0.03	-0.02	-1.22	-1.24	-1.26	-1.28		-0.05	-0.03	-0.01	0.00
up_F	0.36	0.29	0.20	0.11		1.42	2.77	4.05	5.28	0.50	0.40	0.28	0.15		1.32	2.59	3.83	5.03
up_S	0.76	0.78	0.81	0.83		6.84	7.03	7.21	7.39	0.93	0.96	0.99	1.02		7.06	7.22	7.38	7.54
up_B	0.19	0.20	0.21	0.22		1.88	2.01	2.13	2.26	0.19	0.20	0.21	0.21		2.61	2.69	2.76	2.83

Table 3: Portfolio Choice Implications: Asset Allocation and Return Characteristics

The entries of the table are the allocation in stock (S), the allocation in bond (B), the allocation in cash (C), the allocation in the normalized mean-variance portfolio ($\overline{MV}$), the allocation in the normalized asymmetry-variance portfolio ($\overline{AV}$) and the bond/stock allocation ratio (B/S), the mean ($m_F - r_f + \frac{1}{2}\omega_F$), the volatility ($\sqrt{\omega_F}$), the skewness ($s_F$) and the excess kurtosis (κ_F) of the optimal portfolio excess return. Excess risk aversion is fixed to $\alpha = 1$, and loss aversion to $\ell = 2$. The investment horizon is $T = 1$ month. The constant risk-free rate as well as log returns on stock and bond dynamics are calibrated with values reported in the panel GMM C of Table 1. All portfolio weights as well as mean and volatility of excess returns are in monthly percentage values.

		GDA Investor									EU Investor								
κ		0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04
		Optimal Portfolio Weights																	
S	Total	34.80	26.58	18.06	9.22	0.00	18.19	33.98	47.83	60.06	45.28	34.82	23.81	12.21	0.00	13.83	26.59	38.42	49.42
	MV	46.27	35.66	24.43	12.55	0.00	14.22	27.27	39.31	50.47	46.27	35.66	24.43	12.55	0.00	14.22	27.27	39.31	50.47
	AV	-11.47	-9.07	-6.36	-3.34	0.00	3.96	6.71	8.51	9.59	-1.00	-0.83	-0.62	-0.35	0.00	-0.39	-0.68	-0.89	-1.05
B	Total	51.02	39.51	27.19	14.03	0.00	9.52	18.98	28.32	37.45	42.40	32.73	22.46	11.56	0.00	13.10	25.06	36.06	46.21
	MV	41.58	32.04	21.95	11.28	0.00	12.78	24.50	35.32	45.35	41.58	32.04	21.95	11.28	0.00	12.78	24.50	35.32	45.35
	AV	9.44	7.47	5.24	2.75	0.00	-3.26	-5.52	-7.01	-7.89	0.82	0.69	0.51	0.29	0.00	0.32	0.56	0.74	0.87
C		14.18	33.91	54.75	76.76	100.00	72.30	47.04	23.86	2.49	12.33	32.45	53.73	76.23	100.00	73.07	48.35	25.52	4.37
$\overline{MV}$		87.85	67.70	46.37	23.83	0.00	27.00	51.77	74.64	95.82	87.85	67.70	46.37	23.83	0.00	27.00	51.77	74.64	95.82
$\overline{AV}$		-2.03	-1.60	-1.13	-0.59	0.00	0.70	1.19	1.51	1.70	-0.18	-0.15	-0.11	-0.06	0.00	-0.07	-0.12	-0.16	-0.19
B/S		1.466	1.486	1.505	1.522		0.523	0.559	0.592	0.624	0.936	0.940	0.943	0.947		0.947	0.943	0.939	0.935
		Moments of Portfolio Excess Return																	
$m_F - r_f + \frac{1}{2}\omega_F$		0.27	0.20	0.14	0.07	0.00	0.11	0.22	0.31	0.39	0.31	0.24	0.16	0.08	0.00	0.10	0.18	0.26	0.34
$\sqrt{\omega_F}$		1.93	1.48	1.01	0.52	0.00	0.82	1.54	2.18	2.76	2.21	1.70	1.17	0.60	0.00	0.68	1.30	1.88	2.41
s_F		-0.13	-0.13	-0.13	-0.12		-0.42	-0.40	-0.39	-0.38	-0.26	-0.26	-0.26	-0.26		-0.26	-0.26	-0.26	-0.26
κ_F		0.14	0.13	0.13	0.12		0.64	0.61	0.58	0.55	0.34	0.34	0.34	0.34		0.34	0.34	0.34	0.34

Table 4: Portfolio Choice Implications: Loss Probability and Expected Shortfalls

The entries of the table are the loss probability ($\pi^{\mathbf{L}}$), the expected shortfall (es_F) of the optimal portfolio and the associated marginal expected shortfalls of the stock (es_S) and the bond (es_B), the upside potential (up_F) of the optimal portfolio and the associated marginal upside potentials of the stock (up_S) and the bond (up_B). Excess risk aversion is fixed to $\alpha = 1$, and loss aversion to $\ell = 2$. The investment horizon is $T = 1$ month. The constant risk-free rate as well as log returns on stock and bond dynamics are calibrated with values reported in the panel GMM C of Table 1. Expected shortfall and upside potential are in monthly percentage values.

	GDA Investor									EU Investor								
κ	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04	0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04
	Expected Shortfall and Upside Potential, Downside is Negative Excess Return																	
$\pi^{\mathbf{L}}$	43.67	43.70	43.73	43.75		41.91	41.99	42.06	42.13	42.78	42.79	42.80	42.81		42.80	42.80	42.79	42.78
es_F	-1.47	-1.13	-0.77	-0.39		-0.65	-1.21	-1.71	-2.16	-1.71	-1.32	-0.90	-0.46		-0.52	-1.01	-1.45	-1.87
es_S	-2.60	-2.59	-2.57	-2.56		-3.25	-3.23	-3.21	-3.19	-2.97	-2.97	-2.97	-2.97		-2.97	-2.97	-2.97	-2.98
es_B	-1.10	-1.11	-1.12	-1.12		-0.58	-0.61	-0.63	-0.66	-0.86	-0.86	-0.86	-0.86		-0.86	-0.86	-0.86	-0.86
up_F	1.61	1.24	0.85	0.43		0.66	1.25	1.77	2.24	1.82	1.40	0.96	0.49		0.56	1.07	1.55	1.99
up_S	3.00	2.99	2.98	2.98		3.30	3.29	3.28	3.28	3.19	3.19	3.19	3.19		3.19	3.19	3.19	3.19
up_B	1.11	1.12	1.13	1.13		0.67	0.69	0.71	0.73	0.89	0.90	0.90	0.90		0.90	0.90	0.90	0.89
	Expected Shortfall and Upside Potential at 5% Level																	
es_F	-3.87	-2.97	-2.03	-1.04		-1.79	-3.35	-4.73	-5.95	-4.62	-3.56	-2.43	-1.25		-1.41	-2.72	-3.93	-5.05
es_S	-7.98	-7.94	-7.90	-7.86		-9.61	-9.57	-9.54	-9.50	-9.03	-9.02	-9.02	-9.01		-9.01	-9.02	-9.03	-9.03
es_B	-2.15	-2.18	-2.20	-2.23		-0.47	-0.53	-0.60	-0.66	-1.26	-1.27	-1.28	-1.28		-1.28	-1.28	-1.27	-1.26
up_F	4.13	3.17	2.17	1.11		1.66	3.14	4.46	5.64	4.63	3.56	2.44	1.25		1.42	2.72	3.93	5.05
up_S	7.02	7.00	6.98	6.96		7.95	7.92	7.89	7.86	7.56	7.56	7.55	7.55		7.55	7.56	7.56	7.56
up_B	3.30	3.31	3.33	3.34		2.30	2.36	2.41	2.45	2.84	2.85	2.85	2.85		2.85	2.85	2.84	2.84

Table 5: Asset Allocations Recommended by Financial Advisors

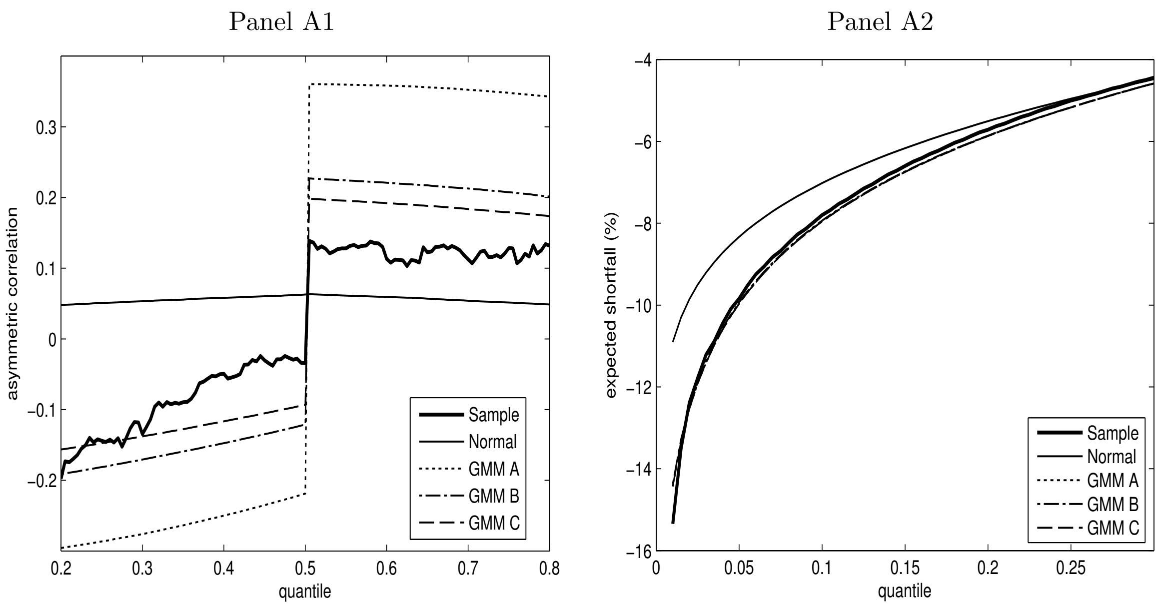
The entries of the table are the recommendations of four financial advisors. The recommendations in part A come from a newsletter sent by Fidelity Investments (Larry Mark, 1993), a large mutual-fund company. Those in part B come from a book promoted by Merrill Lynch (Don Underwood and Paul B. Brown, 1993), a large brokerage firm. Those in part C come from a book by Jane Bryant Quinn (1991), a prominent journalist who writes on personal financial planning. Those in part D come from an article in the "Your Money" section of The New York Times (Mary Rowland, 1994). The last two columns show the effective risk aversion and the implicit asymmetry aversion of conservative, moderate and aggressive investors as perceived by the four financial advisors. The constant risk-free rate as well as log returns on stock and bond dynamics are calibrated with values reported in the panel GMM C of Table 1.

	Percent of Portfolio					
Advisor and investor type	Cash	Bond	Stock	Bond/Stock	γ	χ
A. Fidelity						
Conservative	50	30	20	1.50	10.77	1130
Moderate	20	40	40	1.00	6.86	50
Aggressive	5	30	65	0.46	5.99	-95
B. Merrill Lynch						
Conservative	20	35	45	0.78	6.94	-49
Moderate	5	40	55	0.73	5.87	-47
Aggressive	5	20	75	0.27	6.12	-114
C. Jane Bryant Quinn						
Conservative	50	30	20	1.50	10.77	1130
Moderate	10	40	50	0.80	6.16	-32
Aggressive	0	0	100	0.00	6.08	-110
D. The New York Times						
Conservative	20	40	40	1.00	6.86	50
Moderate	10	30	60	0.50	6.30	-100
Aggressive	0	20	80	0.25	5.83	-104

Table 6: Certainty Equivalent Costs

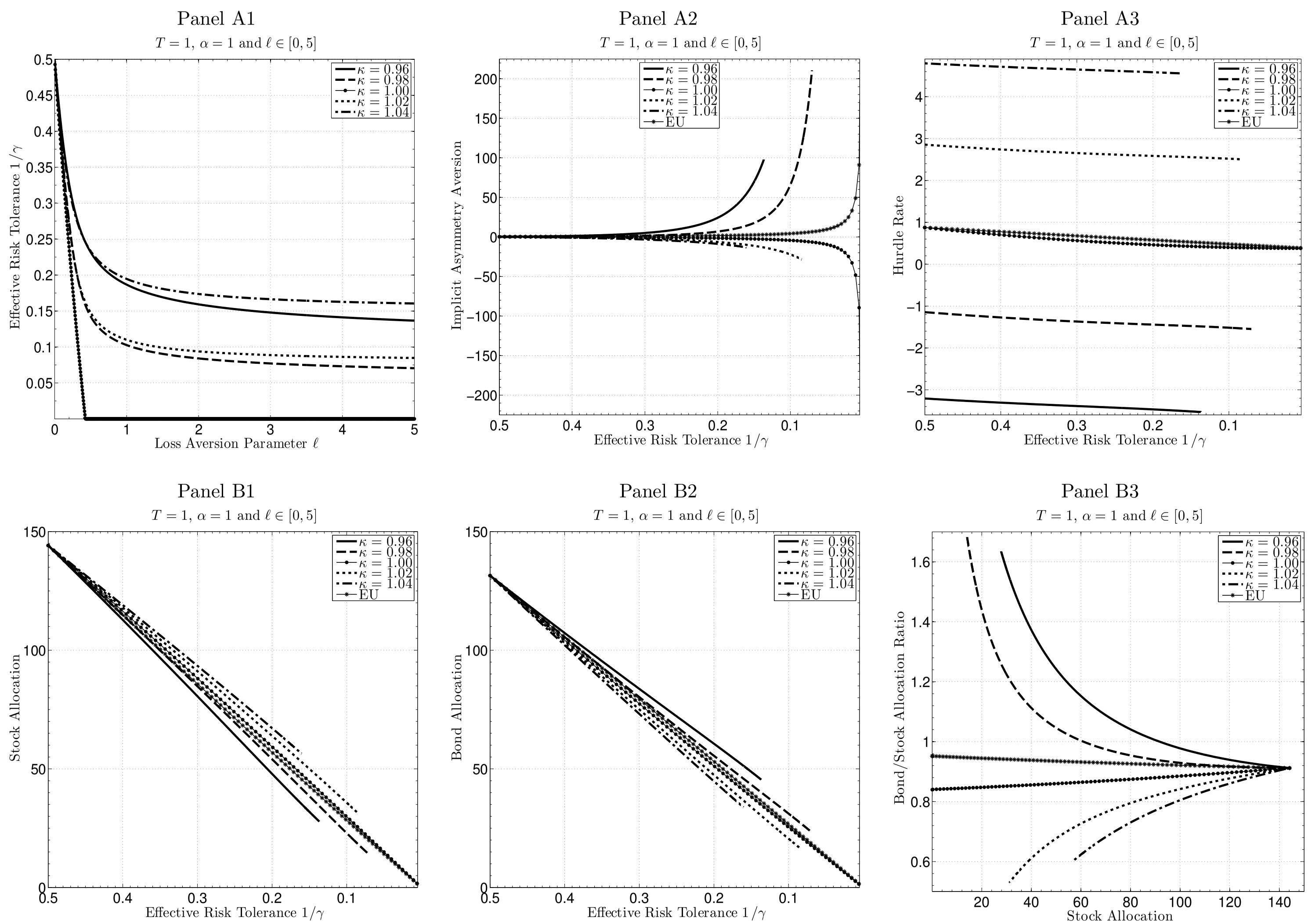
The entries of the table are the allocation in stock (S), the allocation in bond (B), the allocation in cash (C), the certainty equivalent cost of ignoring returns asymmetry ($\hat{\mathcal{R}} - \mathcal{R}$), and the certainty equivalent cost of ignoring preferences asymmetry ($\tilde{\mathcal{R}} - \mathcal{R}$). Excess risk aversion is fixed to $\alpha = 1$, and loss aversion to $\ell = 2$. The investment horizon is $T = 1$ month. The constant risk-free rate as well as log returns on stock and bond dynamics are calibrated with values reported in the panel GMM C of Table 1. All portfolio weights as well as certainty equivalent costs are in monthly percentage values.

κ		0.96	0.97	0.98	0.99	1.00	1.01	1.02	1.03	1.04
		Ignoring Returns Asymmetry								
$\hat{\gamma}$		6.94	9.05	13.28	25.97	∞	18.52	9.81	6.90	5.44
S	Asymmetry	34.80	26.58	18.06	9.22	0.00	18.19	33.98	47.83	60.06
	Normal	41.89	32.10	21.87	11.19	0.00	15.69	29.62	42.11	53.38
	Difference	7.09	5.52	3.81	1.97	0.00	-2.50	-4.35	-5.71	-6.67
B	Asymmetry	51.02	39.51	27.19	14.03	0.00	9.52	18.98	28.32	37.45
	Normal	37.65	28.84	19.66	10.05	0.00	14.10	26.62	37.84	47.97
	Difference	-13.38	-10.67	-7.53	-3.97	0.00	4.58	7.64	9.53	10.52
C	Asymmetry	14.18	33.91	54.75	76.76	100.00	72.30	47.04	23.86	2.49
	Normal	20.46	39.06	58.47	78.76	100.00	70.21	43.76	20.04	-1.36
	Difference	6.28	5.15	3.72	2.00	0.00	-2.09	-3.29	-3.82	-3.84
$\hat{\mathcal{R}} - \mathcal{R}$		-0.0098	-0.0078	-0.0055	-0.0029	0.0000	-0.0015	-0.0024	-0.0029	-0.0031
		Ignoring Preferences Asymmetry								
S	Asymmetric	34.80	26.58	18.06	9.22	0.00	18.19	33.98	47.83	60.06
	Expected Utility	45.28	34.82	23.81	12.21	0.00	13.83	26.59	38.42	49.42
	Difference	10.47	8.24	5.74	2.99	0.00	-4.35	-7.39	-9.41	-10.64
B	Asymmetric	51.02	39.51	27.19	14.03	0.00	9.52	18.98	28.32	37.45
	Expected Utility	42.40	32.73	22.46	11.56	0.00	13.10	25.06	36.06	46.21
	Difference	-8.62	-6.78	-4.73	-2.46	0.00	3.58	6.08	7.74	8.76
C	Asymmetric	14.18	33.91	54.75	76.76	100.00	72.30	47.04	23.86	2.49
	Expected Utility	12.33	32.45	53.73	76.23	100.00	73.07	48.35	25.52	4.37
	Difference	-1.85	-1.46	-1.02	-0.53	0.00	0.77	1.31	1.66	1.88
$\tilde{\mathcal{R}} - \mathcal{R}$		-0.0182	-0.0145	-0.0101	-0.0052	0.0000	-0.0042	-0.0064	-0.0074	-0.0074



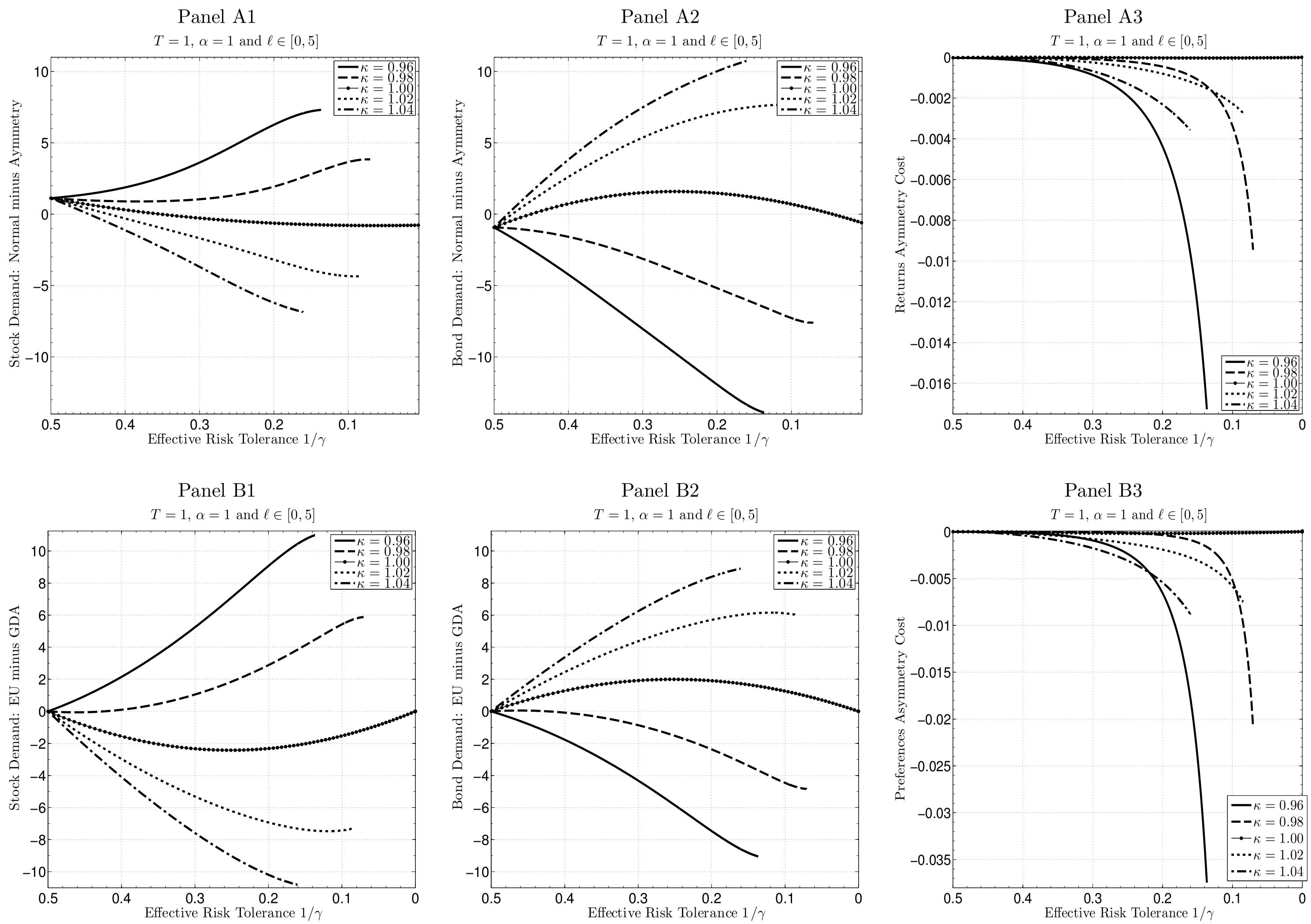
Panel A1 plots asymmetric bond-stock correlations defined as $\begin{cases} Corr\left(r_S, r_B | r_S < Q_S\left(q\right)\right) & \text{if } q \leq 0.5 \\ Corr\left(r_S, r_B | r_S > Q_S\left(q\right)\right) & \text{if } q \leq 0.5 \end{cases}$, where $Q_S\left(q\right)$ denotes the q-th quantile of the stock return distribution. Panel A2 plots the expected shortfall of the stock return defined as $E\left[r_S \mid r_S < Q_S\left(q\right)\right]$ and expressed in monthly percentages (%). Both plots display values estimated from the sample ($Sample$), values simulated from a normal distribution fitted to the data ($Normal$), and values simulated using model (13) fitted to the data using different GMM estimators ($GMM\ A$, $GMM\ B$, and $GMM\ C$).

Figure 1: Asymmetric Bond-Stock Correlations and Stock Expected Shortfall



Panel A1 plots the effective risk tolerance against the coefficient of loss aversion ℓ, for five different values of the GDA parameter $\kappa \in \{0.96, 0.98, 1.00, 1.02, 1.04\}$. Panels A2 and A3 respectively plot the ratio of implicit asymmetry aversion to effective risk aversion, and the hurdle rate, against the effective risk tolerance $1/\gamma$, for the five different values of the GDA parameter and for the EU investor. Panels B1 and B2 show similar plots for the stock allocation and the bond allocation respectively. Panel B3 plots the bond/stock allocation ratio against the stock allocation. The excess regular risk aversion parameter is fixed to $\alpha = 1$ and the loss aversion parameter ℓ varies across 1,000 regularly spaced values in the interval $[0, 5]$. The investment horizon is $T = 1$ month. The constant risk-free rate as well as log returns on stock and bond dynamics are calibrated with values reported in the panel GMM C of Table 1.

Figure 2: Optimal Allocation Policy and Implicit Asymmetry Aversion



Panel A1 plots the difference between stock allocations under asymmetric and normal returns against the effective risk tolerance $1/\gamma$, for five different values of the GDA parameter $\kappa \in \{0.96, 0.98, 1.00, 1.02, 1.04\}$ and for the EU investor. Panel A2 does the same for bond, and Panel A3 plots the certainty equivalent cost of ignoring asymmetry in asset returns. Panel B1 plots the difference between stock allocations under asymmetric and expected utility preferences. Panel B2 does the same for bond, and Panel B3 plots the certainty equivalent cost of ignoring asymmetry in preferences. The excess regular risk aversion parameter is fixed to $\alpha = 1$ and the loss aversion parameter ℓ varies across 1,000 regularly spaced values in the interval $[0, 5]$. The investment horizon is $T = 1$ month. The constant risk-free rate as well as log returns on stock and bond dynamics are calibrated with values reported in the panel GMM C of Table 1. Certainty equivalent costs are expressed in monthly percentage values.

Figure 3: Costs of Ignoring Asymmetries