Ex-ante Estimates of Market Risk Premium Implied from Physical and Risk-Neutral Distributions

by

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Abstract

The paper develops a new methodology to retrieve ex-ante estimates of market risk premium. Assuming the existence of a monotonic projected pricing kernel motivated by the well-known power/logarithmic utility function it first derives a theoretical model that relates physical cumulants of any order to risk-neutral ones through the projected relative risk aversion coefficient. This model implies that the ex-ante market risk premium depends on higher-order physical or risk-neutral cumulants and the projected relative risk aversion coefficient. These higher-order cumulants are related to the conditional volatility, skewness and kurtosis of the physical or risk-neutral distribution. In the empirical part of the paper using stock and option data from the S&P 500 index it estimates a monthly ex-ante market risk premium for the period 2001-2010. The estimated market risk premium is time-varying, counter-cyclical and exhibits high variability during our sample period. Based on these estimates the paper examines the relative importance of expected returns and cash flows shocks in stock price movement without relying on the usual predictive regressions approach. The empirical results indicate that shocks in cash flows is the dominant factor of unexpected return variation.

Keywords: Ex-ante market risk premium, risk aversion coefficient, physical distribution, risk-neutral distribution.

JEL: G12, G17, C51, C53.

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1 Introduction

An asset expected return and its associated risk premium are the most important concepts in finance. To this end, a central issue in asset pricing theory is to model this expected return and, thus, to explain which are the factors that determine it. The expected return is a forward-looking quantity which depends on the systematic risk of the asset as it is perceived ex-ante by investors, albeit it is unobserved by researchers.

In practice, the most commonly used method for estimating an asset expected return and its associated risk premium is by calculating the average value of historical realized returns using a long estimation period. There are at least three major problems with this approach. First, the average realized return is an unconditional estimate only valid under an i.i.d. case. However, a number of studies document that expected returns are time-varying and persistent (see Cochrane (2011), inter alia). Thus, the conditional ex-ante risk premium does not coincide with the ex-post unconditional one. Second, this ex-post estimate that does not take into account changes in market conditions (see Merton (1980)). Third, it is highly probable that the ex-ante risk premium captures a level of risk which is related to the occurrence of some bad states of nature that have not been realized in the sample (the well-known peso problem) (see Brown, Goetzmann and Ross (1995), inter alia).

In order to overcome these deficiencies, three other approaches were proposed in the literature. The first, uses survey on academics, investors or business managers to get their view on risk premium (see Welch (2000) and Graham and Harvey (2007)). The second, employs the present value identity to estimate the implied cost of equity capital from current stock prices and analysts forecasts for future cash flows (see Pastor, Sinha and Swaminathan (2008)). The third approach, combines information from the stock and options market, along with a parametric option pricing model (see Santa-Clara and Yan (2010)) or a non-parametric procedure (see Duan and Zhang (2014)), to estimate the ex-ante risk premium. The first approach is subject to a number of limitations, the most important being that surveys are expressions of subjective opinions and face unknown sample selection bias. The second approach also presents several shortcomings. First, the implied cost of capital even if it conveys information similar to expected returns it does not coincide with the one-period expected return (see Chen, Da and Zhao (2013)). Second, its level is subject to analyst
forecast biases.

This paper develops a new methodology to estimate the market expected return and the associated ex-ante market risk premium, which is in line with the third aforementioned approach. Assuming the existence of a monotonic projected pricing kernel motivated by the well-known power/logarithmic utility function it first derives a theoretical model that relates cumulants of the physical distribution of the market log-return (denoted as physical cumulants) of any order to those of the risk-neutral distribution (denoted as risk-neutral cumulants) through the projected relative risk aversion coefficient. This model implies that the ex-ante market risk premium depends on higher-order physical cumulants and the projected relative risk aversion coefficient. These higher-order physical cumulants are related to the conditional volatility, skewness and kurtosis of the physical distribution. This model indicates that investors require a higher compensation to hold the market portfolio when their aversion towards risk increase and/or when their expectations about future returns include high levels of volatility, negative skewness and excess kurtosis. This model also implies an equivalent formula to estimate the market risk premium in terms of higher-order risk-neutral cumulants. The model also links the projected relative risk aversion coefficient to the difference between higher-order physical and risk-neutral cumulants.

To empirically implement this methodology we need estimates of forward-looking physical and risk-neutral cumulants as well as the projected risk aversion coefficient. Higher-order physical cumulants are retrieved by the Filtered Historical Simulation approach using the S&P 500 daily index returns. We also estimate higher-order risk-neutral cumulants using European option prices written on the S&P 500 index. Then, using a system GMM estimation procedure to the relationships that relate higher-order physical and risk-neutral cumulants through the projected relative risk aversion coefficient, implied by the theoretical model, we estimate this coefficient in a monthly frequency from 2001 to 2010. With this risk aversion coefficient estimated in place, we combine it with higher-order physical and/or risk-neutral cumulants to produce estimates of market expected returns and the associated ex-ante market risk premium for every date of our sample period. Essentially, this method estimates the asset expected return (i.e., the mean of the physical distribution) at every time period so that the dispersion between the physical and risk-neutral distribution can be explained by a monotonic pricing kernel. The estimated market risk premium is always positive, time-varying and counter-cyclical, showing high variability, during the sample pe-
period, from around 1% to around 100%. As expected it increases during the two financial turmoil periods, i.e., the burst of the Dot-Com bubble in 2002 and the sub-prime mortgage crisis (2007 to 2009).

Our approach merits important advantages compared to previous studies that also used stock and option data to estimate the ex-ante market risk premium. In contrast to Santa-Clara and Yan (2010) it does not rely on any parametric option pricing model, as thus it can be considered as model-free. Compared to Duan and Zhang’s (2014) approach, our methodology, even if it is based on the same theoretical assumptions (i.e., the existence of a monotonic pricing kernel that relates physical and risk-neutral distribution), it exploits a larger set of properties of this model. This enables us, first, to produce more accurate estimates of the ex-ante market risk premium and second, to examine in greater depth if the model is correctly specified.

The estimation of a time series of market expected returns enables us to examine a central issue in asset pricing; do stock prices move due to shocks in expected returns or expected cash flows, and how much of each? The prevailing view in the literature is that almost all stock market variation is driven by shocks in expected returns, and almost none by shocks in cash flows (see Cochrane (2001, 2008), *inter alia*). This view is based on the results of predictive regressions, namely that returns have been much easier to predict than dividends in the post-war period. This uncomfortable conclusion implies that, during a depression, where we observe negative shocks to expected cash flows, stock prices would not be affected by them. However, predictive regressions have well-documented shortcomings (see Chen and Zhao (2009), Koijen and Van Nieuwerburgh (2011), Chen, Da and Zhao (2013), *inter alia*). To this end, we suggest an alternative approach, that do not rely on predictability, in order to answer this central question. This approach exploits the estimates of market expected returns retrieved in this paper. In contrast to the prevailing belief, our results indicate that shocks in cash flows is the dominant factor of unexpected return variation for investment horizons larger or equal than 1-month for our sample period. This finding is robust to whether or not the recent financial crisis is included in the sample period or shocks in cash flows are directly modeled or backed out from the observed shocks in expected returns. These results complement an increasing number of recent studies that

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1 As Cochrane (2011) summarized it: "we thought 100% of the variation in the market dividend yields was due to variation in expected cash flows; now we know 100% is due to variation in discount rates."
demonstrate the importance of shocks in cash flows in asset pricing (see Chen, Da and Zhao (2013), *inter alia*).

The paper is organized as follows. Section 2 derives the theoretical model. Section 3 presents the econometric formulation for estimating market expected returns and the associated ex-ante market risk premium. Section 4 describes the data and discuss the estimates of the market risk premium. Section 5 examines the relative importance of shocks in expected returns and expected cash flows in stock price movement, and Section 6 concludes the paper. All of the derivations are given in a technical appendix.

2 Theoretical model

Consider an economy in the absence of arbitrage. Then, there will be a pricing kernel projected onto the market portfolio log-return, denoted as $M_t(r_{t,T})$ with $r_{t,T} = \ln(S_T/S_t)$ where $S_T$ (or $S_t$) denotes the market portfolio price at the future time $T$ (or at the current time $t$), under which the price of the market portfolio at time $t$ is written as:

$$S_t e^{-\delta \tau} = E_t^P[M_t(r_{t,T}) S_T],$$

where $E_t^P[\cdot]$ is the conditional expectation under the physical measure $P$, $\delta$ is the continuously compounded dividend yield of the market portfolio over the period $[t, T]$, assumed to be constant, and $\tau = T - t$. The last relation can be written as:

$$1 = E_t^P\left[M_t(r_{t,T}) e^{r_{t,T} \delta \tau}\right].$$

Assume that $M_t(r_{t,T})$ is implied by a power/logarithmic utility, i.e., $M_t(r_{t,T}) = \beta e^{-\gamma r_{t,T}}$, where $\beta$ is a scaling factor and $\gamma$ is the projected relative risk aversion coefficient.\footnote{The projected pricing kernel implied by a power/logarithmic utility has been employed in many recent studies of the literature to estimate the risk aversion coefficient and the log-return distribution under measure $P$, without specifying any state variable (see, Rosenberg and Engle (2002), Bakshi, Kapadia and Madan (2003), Bliss and Panigirtzoglou (2004), Kang and Kim (2006) and Liu et al (2007), *inter alia*).}

\footnote{This pricing kernel is known as projected pricing kernel (see Cochrane (2001), and Rosenberg and Engle (2002)). Under the assumptions that the asset level is equal to the aggregate wealth and that investors have a finite horizon, the projected pricing kernel is equal to the original pricing kernel (see, e.g. Ait-Sahalia and Lo (2000), Jackwerth (2000) and Bliss and Panigirtzoglou (2004)).}
(2) holds also for a risk-free zero coupon bond that pays $1 at maturity $T$. It is written as:

$$1 = \beta E^P_t \left[ e^{-\gamma r_{t,T}} e^{r_{f,T}} \right],$$  \hspace{1cm} (3)

where $r_f$ is the annualized yield to maturity of the bond at time $t$. Let denote as

$$m^P_t (u) = \ln E^P_t [e^{ur_{t,T}}]$$

the logarithm of the time-$t$ conditional moment generating function of $r_{t,T}$ under measure $P$. Then formula (3) implies that:

$$r_{f,T} = -\ln \beta - m^P_t (-\gamma).$$ \hspace{1cm} (4)

Under the absence of arbitrage there is exist an equivalent martingale measure (also known as the risk-neutral measure $Q$) under which discounted asset prices follow a martingale process. Let define the conditional distribution of $r_{t,T}$ under measure $Q$ as:

$$f^Q_t(r_{t,T}) = e^{r_{f,T}} \beta e^{-\gamma r_{t,T}} f^P_t (r_{t,T}),$$ \hspace{1cm} (5)

where $f^P_t$ is the conditional distribution of $r_{t,T}$ under measure $P$. The new probability measure is correctly specified since $f^Q_t(r_{t,T}) > 0$ and

$$\int f^Q_t(r_{t,T}) dr_{t,T} = e^{r_{f,T}} \beta \int e^{-\gamma r_{t,T}} f^P_t (r_{t,T}) dr_{t,T} = e^{r_{f,T}} \beta e^{m^P_t (-\gamma)} = 1,$$

due to (4). Also one can easily show that under measure $Q$ defined in (5) the discounted asset prices follow a martingale process. Next we write formula (5) in terms of moment-generating functions. This is given as follows:

$$m^Q_t (u) = \ln \beta + r_{f,T} + m^P_t (u - \gamma),$$ \hspace{1cm} (6)

where $m^Q_t (u) = \ln E^Q_t [e^{ur_{t,T}}]$ is the logarithm of the time-$t$ conditional moment generating function of $r_{t,T}$ under measure $Q$. Based on the previous relation we can deduce two equivalent formulas that relate the cumulants of the log-return distribution under the physical and risk-neutral measure. These are given in the following Proposition.

**Proposition 1** Let $k^Q_{r,n}$ and $k^P_{r,n}$ be the $n$th-order cumulants of the $\tau$-period log-return $r_{t,T}$ distribution conditional on the current market information set $\mathcal{I}_t$ under the physical $P$ and
risk-neutral $Q$ measure, respectively. Then, the following relations holds:

$$ k_{r,n}^P = \sum_{m=0}^{\infty} k_{r,n+m}^Q \frac{\gamma^m}{m!}, \quad \forall \ n \in \mathbb{N}, $$

and

$$ k_{r,n}^Q = \sum_{m=0}^{\infty} k_{r,n+m}^P \frac{(-\gamma)^m}{m!}, \quad \forall \ n \in \mathbb{N}. $$

Formula (7) is also proved by Rompolis and Tzavalis (2010). Also, Bakshi, Kapadia and Madan (2003), Bakshi and Madan (2006) and Duan and Zhang (2014) give approximated counterparts of (8) based on variance, skewness and kurtosis coefficients for $n = 3$, $n = 2$ and $n = 1$, respectively. Equation (8) is an exact formula that can be employed for all $n$. To this end, one can view Proposition 1 as refining and generalizing existing results in the literature.

Relations (7) and (8) provide a unified framework to characterize the premia that investors are willing to pay as a compensation towards risk. These premia are manifested in the difference between physical and risk-neutral cumulants. For example, for $n = 1$ formula (8) implies that:

$$ k_{r,1}^P - k_{r,1}^Q = \gamma k_{r,2}^P - \frac{\gamma^2}{2!} k_{r,3}^P + \frac{\gamma^3}{3!} k_{r,4}^P + ... $$

The last formula indicates that the market risk premium, defined as $k_{r,1}^P - k_{r,1}^Q$, is positively related to the physical conditional variance, $k_{r,2}^P$, and excess kurtosis, $k_{r,4}^P$ and negatively related to the skewness of the distribution, measured by $k_{r,3}^P$. This is something to expect. As volatility and the probability of extreme events increase, risk premium also increases. Also, as skewness become more negative, risk premium increases. The effect of these cumulants on risk premium depends on term $\frac{(-\gamma)^m}{m!}$. As expected for $\gamma = 0$ formula (9) implies that $k_{r,1}^P - k_{r,1}^Q = 0$. For small values of risk aversion coefficient, i.e. $\gamma \leq 1$, investors place a larger weight on variance than on skewness and excess kurtosis. As $\gamma$ increases higher-order cumulants effects become more significant for the market risk premium. Formula (9) also indicates that if the log-return $r_{t,T}$ follows the normal distribution, which implies that $k_{r,n}^P = 0$ for $n > 2$, then risk premium can be written as:

$$ k_{r,1}^P - k_{r,1}^Q = \gamma k_{r,2}^P, $$

which a well-known result implied by the CAPM and the Black-Scholes model assumptions.
Formula (9) indicates that knowledge of the projected relative risk aversion coefficient $\gamma$ and the higher-order physical cumulants provides an estimate of the ex-ante market risk premium. Note here that evaluating formula (7) for $n = 1$ gives an equivalent way to estimate this risk premium as:

$$k_{r,1}^P - k_{r,1}^Q = \gamma k_{r,2}^Q + \frac{\gamma^2}{2!} k_{r,3}^Q + \frac{\gamma^3}{3!} k_{r,4}^Q + \ldots \quad (10)$$

To implement the last formula we need an estimate of $\gamma$ and the higher-order risk-neutral cumulants. In the empirical part of the paper we will exploit formulas (9) and (10) to retrieve estimates of the ex-ante market risk premium.4

For $n = 2$ formula (8) gives:

$$k_{r,2}^P - k_{r,2}^Q = \gamma k_{r,3}^P - \frac{\gamma^2}{2!} k_{r,4}^P + \ldots \quad (11)$$

The last formula indicates that the empirically documented difference between physical and risk-neutral variance (see Christensen and Prabhala (1998), *inter alia*) can be attributed to higher than second-order physical cumulants. In case $r_{t,T}$ follows the normal distribution, which implies that $k_{r,n}^P = 0$ for $n > 2$, and/or $\gamma = 0$ then the last formula indicates that $k_{r,2}^P = k_{r,2}^Q$. If however, investors are risk averse and the physical distribution exhibits nonzero skewness and/or excess kurtosis we should observe a difference between physical and risk-neutral variance. Given the empirical evidences that $k_{r,2}^P - k_{r,2}^Q < 0$ (see Christensen and Prabhala (1998), *inter alia*), this can be explained by a negative skewness (i.e., $k_{r,3}^P < 0$)

4Note here that Duan and Zhang (2014) define the market risk premium as $k_{r,1}^P - (r_f - \delta) \tau$. However, since $k_{r,1}^P$ is the mean of the log-return distribution we argue that the corresponding moment under risk-neutral measure $Q$, $k_{r,1}^Q$, should be subtracted in order to appropriately define risk premium. The latter, is the expected continuously compounded return under $Q$. In contrast, $r_f - \delta$ is the annualized expected percentage price change which occurs in a very short period of time.

Given that the risk-neutral mean can be written as:

$$k_{r,1}^Q = (r_f - \delta) \tau - \sum_{m=2}^{\infty} \frac{k_{r,m}^Q}{m!}$$

(see Chalamandaris and Rompolis (2012) for the proof), the ex-ante market risk premium is equal to:

$$k_{r,1}^P - k_{r,1}^Q = (k_{r,1}^P + \delta \tau) - r_f \tau + \sum_{m=2}^{\infty} \frac{k_{r,m}^Q}{m!}.$$ 

The first term in the right-hand-side of the previous formula is the total (with dividends) expected return, while the last term accounts for the convexity adjustment. Due to Jensen’s inequality, it can be easily shown that $k_{r,1}^Q < (r_f - \delta) \tau$, implying that Duan and Zhang’s (2014) estimates are downward biased.
and/or an excess kurtosis (i.e., $k_{r,4}^P > 0$). The difference between physical and risk-neutral variance can also be determined by setting $n = 2$ in formula (7). This gives,

$$k_{r,2}^P - k_{r,2}^Q = \gamma k_{r,3}^Q + \frac{\gamma^2}{2!} k_{r,4}^Q + ...$$  \hspace{1cm} (12)

This formula indicates that the spread between physical and risk-neutral variance is determined by the sign and magnitude of higher than second-order risk-neutral cumulants. As empirically shown by Rompolis and Tzavalis (2010) this negative spread can be explained by the negative skewness (i.e., $k_{r,3}^Q < 0$) of the risk-neutral distribution.

For $n = 3$ formulas (7) and (8) give, respectively,

$$k_{r,3}^P - k_{r,3}^Q = \gamma k_{r,4}^Q + ...$$  \hspace{1cm} (13)

and

$$k_{r,3}^P - k_{r,3}^Q = \gamma k_{r,4}^P + ...$$  \hspace{1cm} (14)

Both formulas indicate that as soon as $k_{r,4}^Q > 0$ and $k_{r,4}^P > 0$ (i.e., log-return distribution exhibits leptokurtosis for both measures) then the third-order cumulant under the physical measure is larger in magnitude compared to the risk-neutral counterpart. This implies that risk-neutral density is more negatively skewed compared to the physical one. Using similar arguments for $n > 3$ formulas (7) and (8) can explain the difference between $n$th-order physical and risk-neutral cumulants in terms of the projected relative risk aversion coefficient and higher than $n$th-order physical or risk-neutral cumulants.

3 Econometric formulation

As already discussed in order to implement formulas (9) and (10) in order to get ex-ante estimates of market risk premium we first need to estimate the relative risk aversion coefficient $\gamma$ and the higher than first-order physical and risk-neutral cumulants. In this section we will describe the econometric formulation employed by the paper to estimate these variables.

3.1 Estimating higher-order physical cumulants

We estimate the cumulants of the physical distribution at any point in time using the Filtered Historical Simulation (FHS) method of Barone-Adesi, Bourgoin and Giannopoulos...
(1998). The method aims at the construction of the forward-looking physical distribution by combining information from a calibrated conditional variance model and from the empirical distribution of the innovations that drive the return process.

Let’s denote as $\tau_t$ the 1-day log-return of the market portfolio. We assume a general ARMA process $\nu_t$, i.e.

$$\tau_t = \nu_t + \alpha_t,$$

where $\alpha_t = \sigma_t \varepsilon_t$ is conditionally heteroskedastic following mean-reverting, stochastic volatility dynamics. In that sense, $\varepsilon_t$ is a general white noise.

The first stage of the method is a straightforward, if rather cumbersome, econometric exercise of identifying the joint ARMA / variance model $\sigma_t$ that best explains the conditional mean and variance dynamics of the return process. At the second stage, the fitted model serves as a filter that helps extract the primitive innovations $\varepsilon_t$ that drove the process in its observed history. The strength of the method lies in the fact that the filtered innovations $\varepsilon_t$ do not follow a specific form of randomness: It is exactly because of the particularities in the density of these innovations that the empirical distribution differs from standard parametric norms. At the last stage the forward-looking empirical physical distribution is reproduced by simulating the calibrated conditional variance model $\sigma_t$ on random paths staring from the current date $t$ and moving forward using as random updates bootstraps from the filtered innovations set $\varepsilon_t$. This combination of parametric variance filters with the non-parametric innovations’ distribution is the main reason why the method is characterized as "semi-parametric" in the literature (see Barone-Adesi, Engle and Mancini (2008)).

For the purposes of our analysis, we repeat the method for each data point in our sample. On each of these points, we chose the best model among the “adequate”\(^5\) ones based on the Bayesian Information Criterion and using a moving window that spanned the 500 observations $\{\tau_{t-499}, \tau_{t-498}, \ldots, \tau_t\}$ that preceded our current date.

The candidate models range from the simpler GARCH specifications (see Bollerslev (1986)) that address the volatility clustering effect of the observed returns, to the more elaborate variations of Nelson’s (1991) EGARCH and Glosten, Jagannathan and Runkle’s (1993) Threshold - GARCH (or GJR) model that in addition account for asymmetries in the

\(^5\) A model is characterized as “adequate” if its standardized residuals exhibit neither autocorrelation nor heteroskedasticity. We tested for the existence of the first using the Q-test and for the existence of the second using the ARCH-test.
return distribution. The candidate model set completed the GARCH-In-Mean specification that associates the conditional mean return with its conditional volatility. The total number of models considered each time is 648, half of which assumed normal deviates in the log-likelihood function and the other half \(t\)-Student updates.

It is very interesting that in almost 93% of our data points, the algorithm converge to the selection of an ARMA(0,1) specification with the conditional variance described by an EGARCH(2,2)

\[
r_t = a_t + \theta \cdot a_{t-1}
\]

\[
\ln \left( \sigma_t^2 \right) = \gamma_0 + \sum_{i=1}^{2} \gamma_i |a_{t-i}| + \delta_i \cdot a_{t-i} \frac{\sigma_{t-i}}{\sigma_{t-i}} + \sum_{i=1}^{2} \beta_i \ln \left( \sigma_{t-i}^2 \right).
\]

Other specifications that also prevail on only a few given months include ARMA(1,1) and ARMA(0,0) mean models with variance described by either EGARCH(2,2), EGARCH(1,3) and EGARCH(2,3).

We use a moving window of 500 daily returns to balance the trade-off between having a sufficiently large sample to estimate the variance model and still retain only the most recent history so as not to produce an “averaged” empirical distribution. We also want to keep in line with the prevailing Risk Management practice of using a history of about 2 years to calculate Value at Risk (e.g. RiskMetrics).

Having estimated a set of 500 innovations for each data point of our analysis, we then proceed to the simulation stage. Using \(N = 100,000\) bootstraps randomly chosen (with replacement) from the filtered innovation set, we generate an equal number of paths that took the return from its observed current value \(r_t\) to its new random value \(r_{t,t+\tau}^{(i)}\), for the \(i\)th path, at the end of the \(\tau\)-period that we consider. The time-aggregation on each path provided us with 100,000 simulated \(\tau\)-period log-returns of which we could estimate the cumulants of their distribution.\(^6\)

The main benefit of our approach as opposed to using a single filter specification throughout the entire sample is that we produce physical distributions whose cumulants evolve in a much smoother way. Indeed, if we were to calibrate the same “average” model for all

\(^{6}\)Starting from the \(n\)th-order non-central moment estimates \(\mu_{r,n}^P = \frac{1}{N} \sum_{i=1}^{N} \left( r_{t,t+\tau}^{(i)} \right)^n\) we calculate the \(n\)th-order physical cumulant \(k_{r,n}^P\) by applying the well-known \(k\)-Statistics estimators.
the data points in our sample, on those data points where the “average” specification was to be inferior to the “locally optimal” volatility model, the filtered innovations would have to be by construction larger in magnitude so as to compensate for the regime change in the return dynamics. By allowing structural flexibility in the filter specification, we avoid outliers in the filtered innovations, which in turn would contaminate the inferred empirical distribution. In empirical tests that we do not present here for the sake of brevity, but which are available from the authors upon request, we demonstrate that the use of the single specification results in physical cumulants that are much more noisy than the ones we produced.

3.2 Retrieving risk-neutral cumulants from option prices

Ex-ante estimates of the non-central moments of the risk-neutral density \( \mu^Q_{r,n} \), denoted as the risk-neutral moments, can be directly obtained from out-of-the-money (OTM) European call and put prices employing the formulas suggested by Bakshi, Kapadia and Madan (2003) for \( n = 1, 2, 3, 4 \) and, recently, extended by Rompolis and Tzavalis (2008) to any order \( n \) as

\[
\mu^Q_{r,1} = e^{(r_f - \delta)\tau} - 1 - e^{r_f \tau} \left[ \int_{S_t}^{+\infty} \frac{1}{K^2} C_t(\tau, K) dK + \int_{0}^{S_t} \frac{1}{K^2} P_t(\tau, K) dK \right], \text{ for } n = 1,
\]

\[
\mu^Q_{r,n} = e^{r_f \tau} \left\{ \int_{S_t}^{+\infty} \frac{n}{K^2} \left[ \ln \left( \frac{K}{S_t} \right) \right]^{n-2} \left[ n - 1 - \ln \left( \frac{K}{S_t} \right) \right] C_t(\tau, K) dK + \int_{0}^{S_t} \frac{n}{K^2} \left[ \ln \left( \frac{K}{S_t} \right) \right]^{n-2} \left[ n - 1 - \ln \left( \frac{K}{S_t} \right) \right] P_t(\tau, K) dK \right\}, \text{ for } n \geq 2.
\]

where \( C_t(\tau, K) \) and \( P_t(\tau, K) \) denote the European call and put option prices with strike price \( K \) and maturity interval \( \tau = T - t \). As before \( r_f \) denotes the annual return of the risk-free asset and \( \delta \) denotes the continuously compounded dividend yield. As these formulas employ integrals of continuous functions to retrieve the values of the risk-neutral moments based on them, we can employ cubic splines to interpolate the implied by our option prices volatilities between two different points of the data. Due to the lack of option prices at 0 and \(+\infty\), we can extrapolate the implied volatilities constantly over the intervals \([0, K_{\min}]\) and \([K_{\max}, +\infty)\), where \( K_{\min} \) and \( K_{\max} \) is the minimum and maximum strike prices given by our data, respectively. The extrapolation is truncated at strike prices, denoted as \( K_0 \) and \( K_\infty \), which correspond to put and call prices that are virtually zero (e.g. smaller than \( 10^{-3} \)). These strike prices, define the lower and upper bounds of the integrals, respectively.
Ex-ante risk-neutral cumulants $k_{r,n}^{Q}$ can be obtained using the $k$-Statistics estimators in accordance with the non-central moment estimates given by (15).

### 3.3 Estimating the projected relative risk aversion coefficient

The next step of our procedure is to estimate the projected relative risk aversion coefficient. To this end, we can rely on the results of Proposition 1. More specifically, formulas (7) or (8) for $n > 1$ can form a system of equations which along with the estimates of higher than first-order physical and risk-neutral cumulants can provides us with an estimate of $\gamma$.

Following Bakshi and Madan (2006) we rely on a GMM estimation procedure. Let $I_t$ be a set of instruments whose values are known at time $t$. Then a GMM estimation can be performed using the following orthogonality conditions implied by formula (7):

$$
E \left[ k_N^{P,Q} - \sum_{m=1}^{M} k_{m+2,m+N}^{Q} \frac{\gamma_m^m}{m!} I_t \right] = 0, \tag{16}
$$

where $k_N^{P,Q} = (k_{r,2}^P - k_{r,2}^Q, \ldots, k_{r,N}^P - k_{r,N}^Q)'$ and $k_{m+2,m+N}^{Q} = (k_{r,m+2}^Q, \ldots, k_{r,m+N}^Q)'$. Similarly, the orthogonality conditions implied by (8) can be written as:

$$
E \left[ k_N^{P,Q} + \sum_{m=1}^{M} k_{m+2,m+N}^{P} \frac{(-\gamma)^m}{m!} I_t \right] = 0, \tag{17}
$$

where $k_{m+2,m+N}^{P} = (k_{r,m+2}^P, \ldots, k_{r,m+N}^P)'$.

Bakshi and Madan (2006) and Duan and Zhang (2014) used (17) for $N = 2$ and $M = 2$ (i.e., a second-order approximation of formula (11)) to estimate the relative risk aversion coefficient. For the same purpose, Rompolis and Tzavalis (2010) employed (16) for $N = 2$ and $M = 2$ (i.e., a second-order approximation of formula (12)). Both of these approaches provide a single-equation estimation of $\gamma$ which only accounts for the difference between the physical and risk-neutral variance as this is explained by physical or risk-neutral skewness and excess kurtosis. We generalize this approach in two fronts. First, we increase the number of regressors $M$ in (16) and (17). Theoretically, this will enables us to retrieve a consistent estimator of $\gamma$ with a weaker set of instruments $I_t$. Second, we increase the number of equations $N$ in (16) and (17). This will introduce into the GMM estimation procedure a richer set of information concerning the dispersion between physical and risk-
neutral density, thus, providing a more efficient estimator for $\gamma$.

4 Empirical analysis

In this section, using the aforementioned econometric formulation and data from the S&P 500 index for the period 1996-2010, we estimate the ex-ante market risk premium for a 1-month investment horizon.

4.1 The data

Our empirical analysis uses index daily returns and cross-section sets on European option prices for the S&P 500 index. We consider closing prices of the at-the-money (ATM) and out-of-the money (OTM) put (i.e. $K < S_t$) and call (i.e. $K \geq S_t$) options for the third Wednesday of each month from January 1996 to October 2010. To address liquidity and quality issues in the prices of deep-OTM options we take a number of precautionary steps in the data pre-processing stage of our analysis. That is, to select our final sample, we apply several data filter. First, option quotes less than $3/8$ are excluded from the sample. These prices may not reflect true option value due to proximity to tick size. Second, we have excluded options contracts that had zero trading volume and/or open interest. Third, options violating the boundary conditions are eliminated from the sample. The purpose of these filters is to make our risk-neutral cumulants estimates as insensitive as possible to market microstructure and liquidity biases.

In that context, we also exclude in-the-money (ITM) options because they are less actively traded than ATM and OTM counterparts. Furthermore, risk-neutral moment formulas (see equation (15)) are expressed in terms of ATM and OTM call and put options only. The option data are downloaded from OptionMetrics Ivy DB. The maturity interval of these option prices is approximately equal to one calendar month (i.e. roughly twenty two trading days). Hence, the estimates of risk-neutral moments derived by the above option data sets have the key feature that they result in nonoverlapping observations, as time periods covered by successive options exhibit no overlap.

As the risk-free interest rate and dividend yield we use the estimates employed in the OptionMetrics calculations. The interest rate is derived from British Banker’s Association LIBOR rates and settlement prices of Chicago Mercantile Exchange Eurodollar futures.
The dividend yield is estimated by the put-call parity relation of ATM option contracts (see also Ait-Sahalia and Lo (1998)). We refer the reader to the Ivy DB reference manual for further details.

4.2 Physical and risk-neutral cumulants estimates

Table 1 reports summary statistics for physical and risk-neutral cumulants monthly estimates used in the estimation procedure. For expositional reasons we present the descriptive statistics of second (i.e., the variance), third and fourth-order cumulants. Note here, that in the estimation of $\gamma$ we use physical and risk-neutral cumulants up to order 10. Figure 1 graphically presents the time series estimates of these physical and risk-neutral cumulants. The results of the table and the figure both indicate that the risk-neutral variance is generally higher than the physical one, which is consistent with previous findings in the literature. Third-order cumulant estimates, which account for the skewness of the distribution, are negative under both measures. However, third-order cumulant estimates of risk-neutral distribution are generally higher in absolute terms than the corresponding physical ones. This indicates that the risk-neutral distribution is more negatively skewed than the physical one. Fourth-order cumulant estimates are positive both for physical and risk-neutral distribution with the fourth-order cumulant estimates of risk-neutral distribution being generally higher than the corresponding physical ones. This indicates that the risk-neutral distribution exhibits fatter tails than the physical one. The correlation coefficient between respective physical and risk-neutral cumulants is very high and decreases with respect to the cumulant order. Thus, physical and risk-neutral distributions are related to each others. Any shift in the values of physical distribution would be also reflected, through a pricing kernel, to the values of the risk-neutral ones. The plots of Figure 1 also indicate a significant time variation in the values of higher-order cumulants. Inspection of this figure indicate a shift in the values of risk-neutral and physical cumulants during periods of financial crises (e.g. the Asian currency crisis, the Russian default, the burst of the Dot-Com bubble and Lehmans’ Brothers default) or unexpected events (e.g. the attack on the World Trade Center on 9/11).

The average values of physical and risk-neutral cumulants estimates reported in Table 1 are in accordance with the predictions of the theoretical model presented in Proposition 1. The observed negative difference between physical and risk-neutral variance can be
explained by the negative physical or risk-neutral third-order cumulant estimates and the positive physical or risk-neutral fourth-order cumulant estimates, according to equations (11) or (12), respectively. Similarly, the positive difference between physical and risk-neutral third-order cumulant can be explained by the positive physical or risk-neutral fourth-order cumulant estimates, according to equations (13) or (14), respectively.

4.3 Projected relative risk aversion coefficient estimates

To estimate the projected relative risk aversion coefficient $\gamma$ we employ the orthogonality conditions (17) for $M = 2$ to 5 and $N = 2$ to 5. For $N = 2$ and $M = 2$, we estimate $\gamma$ using a single equation setup which models the difference between physical and risk-neutral variance.\(^7\) This estimation approach is similar to that proposed by Bakshi and Madan (2006) and Duan and Zhang (2014). We generalize this procedure by sequentially increasing both the number of regressors and the number of equations in the system, setting $M = 3, 4$ and 5 and $N = 3, 4$ and 5, respectively. For example for $N = 2$ and $M = 5$ we estimate $\gamma$ by a single equation which, apart from $k_{r,3}^P$ and $k_{r,4}^P$, also attributes the spread between physical and risk-neutral variance to higher than fourth-order physical cumulants.\(^8\) For $N = 3$ and $M = 2$ we estimate $\gamma$ by a two-equations system, which also accounts for the difference between physical and risk-neutral third-order cumulant.\(^9\) For robustness, we also estimate $\gamma$ by employing the orthogonality conditions (16) for $M = 2$ to 5 and $N = 2$ to 5. If the theoretical model is correctly specified then the estimates of $\gamma$ should be similar under both econometric specifications (i.e., (16) and (17)).

A number of studies (see Rosenberg and Engle (2002) and Bliss and Panigirtzoglou

\(^7\)Formally, the orthogonality condition is written as

$$E \left[ k_{r,2}^P - k_{r,2}^Q - \gamma k_{r,3}^P + \frac{\gamma^2}{2!} k_{r,4}^P \right] = 0$$

which can be considered as a second-order approximation of formula (11).

\(^8\)Formally, the orthogonality condition is written as

$$E \left[ k_{r,2}^P - k_{r,2}^Q + \sum_{m=1}^{5} \frac{(-\gamma)^m}{m!} k_{r,m+2}^P \right] = 0$$

which can be considered as a fifth-order approximation of formula (11).

\(^9\)Formally, the orthogonality conditions are written as

$$E \left[ \begin{array}{c} k_{r,2}^P - k_{r,2}^Q - \gamma k_{r,3}^P + \frac{\gamma^2}{2!} k_{r,4}^P \\ k_{r,3}^P - k_{r,3}^Q - \gamma k_{r,4}^P + \frac{\gamma^2}{2!} k_{r,5}^P \end{array} \right] = 0$$

which can be considered as a second-order approximation of formulas (11) and (14).
(2004)) provided empirical evidences that the projected relative risk aversion coefficient is
time-varying and also related to business cycle indicators. To this end, we estimate $\gamma$ using
a 6-year moving window of data updated monthly. More specifically, the estimate of $\gamma$ at
the observation date $t$ is obtained using 6-year data prior to and including $t$ to generate
72 monthly time series of the relevant variables for the GMM estimation.\(^{10}\) Note here that
because of the 6-year rolling window of data used, the estimation period for $\gamma$ span the
time period from December 2001 to October 2010. Three sets of instruments are used in
the GMM estimation procedure for (17). The first contains the constant and one period
lagged values of $k^{Q}_{r,2}$, $k^{Q}_{r,3}$, $k^{Q}_{r,4}$ and $k^{Q}_{r,5}$ for the first, second, third and fourth equation
of the system, respectively. The second and third set of instruments also use two and two
and three periods of lagged values of the same variables, respectively. For (16) we use a
similar structure for the three sets of instruments where the risk-neutral cumulants are now
replaced with the respective physical ones. The estimation results from the three sets of
instruments are similar for both econometric specifications, therefore we only report those
from using the third one.

Table 2 reports descriptive statistics of the estimation results, across the sample period,
for both specifications, (16) and (17) and for different values of $N$ and $M$. More precisely, it
reports the average, standard deviation, minimum and maximum values of the estimates of
$\gamma$. It also reports the average standard error and the average and minimum values of $p$-value
of Hansen’s overidentified restriction test. If the null hypothesis of this test is rejected then
this will indicate that the estimated model is misspecified. The source of misspecification
can be alternative class of pricing kernels, the number of equations in the system, or the
order of approximation of the theoretical formulas (7) and (8).

Several conclusions can be drawn from the results of Table 2. First, the estimates of $\gamma$, for
all different specification examined, are intuitively sensible, statistically significant and
comparable with the results of other recent studies that estimated $\gamma$ jointly from stock and
option data.\(^{11}\) Second, these results indicate that the inclusion of extra regressors (i.e., by

\(^{10}\)We have also performed the GMM estimation procedure using different sizes of moving window of data.
These preliminary results indicate that the 6-year moving window provides a smooth series of time-varying
estimates of $\gamma$ for both econometric specifications with the smallest number of time series observations. Even
when we used a larger size moving window of data, the estimates of $\gamma$ are similar to those reported in the paper.

\(^{11}\)Several recent papers have estimated $\gamma$ using stock and options data from the S&P 500 index. For
example, Rosenberg and Engle (2002) reported an estimate of $\gamma$ close to 7, Bliss and Panigirtzoglou (2004)
reported an estimate close to 4, Liu et al (2007) estimated $\gamma$ to be 1.8, Rompolis and Tzavalis (2010) reported
increasing $M$) decreases the estimates of $\gamma$ and its standard error. This is true for all $N$ and for both specifications examined, with the exception of $M = 5$ for (16). This result indicates that lower-order approximations of the theoretical formulas may produce inconsistent estimates of the projected relative risk aversion coefficient. This is due to the fact that the instruments used may not be orthogonal to the error term, which includes lower-order physical or risk-neutral cumulants. The decrease in standard errors with respect to $M$, indicates that the additional regressors have explanatory power on the dependent variables.

Third, the inclusion of new equations (i.e., increasing $N$) in the GMM estimation procedure, decreases standard errors. This is true for all $M$ and for both specifications examined. As already argued, this is something to expect, since the inclusion of new equations introduces into the estimation procedure a richer set of information concerning the dispersion between physical and risk-neutral distribution, thus, providing a more efficient estimator for $\gamma$. Fourth, the overidentified restrictions afforded by the modeling structure is rejected for several observation dates at the 5% significance level for small numbers of $N$. However, for $N \geq 4$ (i.e., for at least a three-equation system) these restrictions are not rejected across all observation dates. This provides evidence that the model constitutes a correct specification of the data. Overall, the results of Table 2 indicate that the accurate estimation of $\gamma$ under the theoretical models (7) and (8) requires a higher-order approximation of the series expansions and a larger number of equations in the estimation procedure than the ones used by Bakshi and Madan (2006) and Duan and Zhang (2014). Finally, the results of the table also indicate that the estimates of $\gamma$ are robust to the selected specification of the theoretical model, i.e., (16) and (17). This provides further evidence that this model is correctly specified.

As an illustration, Figure 2 plots the estimates of $\gamma$ across the sample period for three different specifications. The first is the one used by Bakshi and Madan (2006) and Duan and Zhang (2014) (i.e., (17) for $N = 2$ and $M = 2$). The second and third specification estimates $\gamma$ setting $N = 4$ and $M = 4$ in (17) and (16), respectively. Inspection of the figure leads to several interesting conclusions. First, as already reported in the literature, the projected relative risk aversion coefficient varies smoothly across time. Second, all three

\footnotesize
\begin{itemize}
\item an estimate close to 1.3, Rompolis (2010) estimated $\gamma$ to be close to 0.5 and Duan and Zhang (2014) reported an estimate close to 4.
\item Similar graphs are generated from the other estimated specifications of the model.
\end{itemize}
estimated series move in conjunction. However, as already noted earlier, a higher-order approximation specification which uses a system of equations to estimate \( \gamma \) provides lower, and more stable, estimates of it. Third, for \( N = 4 \) and \( M = 4 \) the estimates of \( \gamma \) are close to each for both econometric specifications, (16) and (17). The highest value for \( \gamma \) in our sample period is observed on September 2007 for all three specifications. Not by coincidence, this period coincides with the beginning of the recent financial crisis. Thus, when investors realized that a period of expansion has ended and the economy will move into recession, they became more risk averse.\(^{13}\) From this period onward we observe a sharp decrease in the estimates of \( \gamma \) reaching its lowest value (i.e., close to 1.5) on October 2008, i.e., at the period following Lehmans’ Brothers default. This positive investors reaction could be due to the emergency government programs announced and implemented during September-October 2008 to assist the financial sector. These include the temporary guarantee program of the US Treasury for money market funds, the Fed program to lend against high-quality asset-backed commercial papers, and most importantly the Troubled Asset Relief Program (TARP). After this period, \( \gamma \) increases sharply close to 2.5 and remains constant until the end of the sample period.

4.4 Ex-ante market risk premium estimates

Using the estimates of \( \gamma \) along with higher-order physical and risk-neutral cumulants we can compute ex-ante estimates of the S&P 500 index risk premium for each observation date for an investment horizon of 1-month. To do so, we employ a third-order approximation of formula (9) and (10).\(^{14}\) Table 3 reports sample descriptive statistics of annualized ex-ante market risk premium estimates, \( MRP \equiv 12 \left( k_{r,1}^P - k_{r,1}^Q \right) \), for the different model specifications employed in order to estimate the relative risk aversion coefficient. Figure 3 plots the time series of these estimates for the three specifications used in Figure 2.

\(^{13}\)In terms of the theoretical model this is due to the fact that the increase in the magnitude of risk-neutral cumulants on September 2007 could not be explained by an analogous increase of the physical ones.

\(^{14}\)These approximations are given as:

\[
k_{r,1}^P - k_{r,1}^Q \simeq \gamma k_{r,2}^P - \frac{\gamma^2}{2!} k_{r,3}^P + \frac{\gamma^3}{3!} k_{r,4}^P
\]

and

\[
k_{r,1}^P - k_{r,1}^Q \simeq \gamma k_{r,2}^Q + \frac{\gamma^2}{2!} k_{r,3}^Q + \frac{\gamma^3}{3!} k_{r,4}^Q,
\]

respectively. Note here that the inclusion of higher than fourth-order physical or risk-neutral cumulants in the above expansions has a marginal effect on the estimates of the market risk premium.
Several conclusions can be drawn from the results of Table 3 and Figure 3. First, the MRP is always positive, time-varying and counter-cyclical, showing high variability, during the sample period, from around 1% to around 100%. As expected it increases during the two financial turmoil periods, i.e., the burst of the Dot-Com bubble in 2002 and the sub-prime mortgage crisis (2007 to 2009). During the later period we observe a steady increase of the MRP during the second semester of 2007. This is attributed to the high values of $\gamma$ during this period and the steady increase of physical and risk-neutral cumulants. Its highest value, in our sample, is observed on November 2008 (around 100%). This value is now due to extreme values of higher-order physical and risk-neutral cumulants, $k_{r,j}^P$, $k_{r,j}^Q$ for $j = 2, 3, 4$ (see Figure 1). Thus, during the recent financial crisis changes in the market risk premium were due to different factors. In 2007 MRP increased because investors mainly increased their aversion towards risk, while in 2008 it increased because investors anticipated an increasing uncertainty in the market. Second, the first-order autocorrelation, denoted as $R(-1)$, of the MRP is highly positive indicating the persistence of it. Third, reflecting the estimates of $\gamma$ reported in Table 2, we observe that the values and variability of the MRP decreases as $M$ and/or $N$ increase. Also, the estimates of the MRP obtained by applying formulas (9) and (10) are close to each other, providing evidence that the theoretical model is consistent with the data.

The average, across our sample, MRP which is close to 16% can give us an estimate of the unconditional market risk premium. It worth comparing this estimate with that implied by the realized levels of higher-order physical cumulants. Using sample monthly returns from 2001 to 2010 we estimate a realized level of variance equal to 0.0022 and a realized level of third and fourth-order cumulants equal to -0.00008 and 0.000006, respectively. These estimates are significantly lower in magnitude compared to the average levels of ex-ante physical cumulants reported in Table 1. For example, the sample third-order cumulant is 68% higher than the average level of the ex-ante third-order cumulant. Using these sample statistics and a third-order approximation of formula (9), we obtain an unconditional market risk premium close to 8%. This estimate approximately matches the historical average market risk premium of between 4% and 9% (depending on the sample period) reported by

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15 This unconditional risk premium relies on an ex-post estimate of $\gamma$ equal to 2.74. This is retrieved by employing the previous econometric framework, i.e, orthogonality conditions (17) for $M = 2$ and $N = 4$, for the full sample of physical and risk-neutral cumulants observations. This estimate of $\gamma$ is robust to different econometric specifications of the model.
Thus, the unconditional market risk premium obtained from ex-ante cumulant estimates is close to twice that obtained from ex-post ones. This result provides evidence that a significant part of the ex-ante market risk premium is due to a level of risk which, even if it priced in the market, it is not materialized in the sample (the well-known peso problem). This level of risk, which is embedded in option prices (which is reflected in the risk-neutral cumulants) and in the innovations used in the simulation of market log-returns (which is reflected in the ex-ante physical cumulants), far exceeds the realized variation in stock market returns. This result is in accordance with previous studies (see Brown, Goetzmann and Ross (1995), Rietz (1988) and Santa-Clara and Yan (2010), inter alia) which have pointed out that ex-ante risk premiums capture a level of risk which is related to the occurrence of some bad states of nature that have not been realized in the sample.

5 What drives stock price movement?

In this section we examine the relative importance of shocks in expected returns and expected cash flows in stock price movement. To this end, we use the estimates of market expected returns reported in the previous section. The analysis is conducted in two steps. First, we fit a time-series model in the expected return data. This model, along with the law of iterated expectations, enables us to estimate the term structure of expected returns for each day of our sample. In the second step, we use this term structure to conduct the return decomposition procedure, for different investment horizons.

5.1 The term structure of expected returns

For the reminder of the paper we choose the estimates of expected returns provided by specification (17) with $N = M = 4$. The results are robust to different estimates reported in Table 3. Let $r_{t,t+i} = \ln (S_{t+i}/S_{t})$ denotes the log-return without dividends between dates $t$ and $t + i$, where $i = 1, 2, \ldots$ months. Then, define as $x_t \equiv k_{r,1} = E_t^P [r_{t,t+1}]$, i.e., the one-month ahead expected return estimated at time $t$. By examining the properties of $x_t$ (autocorrelation and partial autocorrelation functions) we conclude that $x_t$ can be modeled
as an AR(1) process:\(^{16}\)

\[ x_{t+1} = \mu + \varphi x_t + \varepsilon_{t+1}, \]  

where \( \varepsilon_{t+1} \sim WN(0, \sigma^2) \). The least squares estimates of this model, with standard errors in parentheses, are given as follows:

\[ \hat{x}_{t+1} = 0.0024 + 0.77 x_t, \quad \hat{\sigma} = 0.006, \quad R^2 = 59\%. \]  

(19)

As already noted in Table 3, we observe that \( x_t \) is a persistent process. Under model (18) we can directly obtain the annualized term structure of expected returns. This is given in the next proposition.

**Proposition 2** If \( x_t \sim AR(1) \) model (18) then,

\[ \frac{1}{(j/12)} E_t^P [r_{t,t+j}] = \frac{12\mu}{1 - \varphi} + \frac{1}{(j/12)} \frac{1 - \varphi}{1 - \varphi} \left( x_t - \frac{\mu}{1 - \varphi} \right), \]  

for \( j = 1, 2, \ldots \) months. Also, the long-term annualized expected return is given as:

\[ \lim_{j \to \infty} \frac{1}{(j/12)} E_t^P [r_{t,t+j}] = \frac{12\mu}{1 - \varphi}. \]  

(21)

The time-invariance of the long-term annualized expected return, given by formula (21), is a direct implication of the stationarity of \( x_t \). Given the estimates reported in (19), this is equal to 12.6\%. Formula (20) implies that the shape of the term structure at time \( t \) depends on the difference between the value of the 1-month ahead expected return \( x_t \) and its long-run mean. If \( x_t < \frac{\mu}{1 - \varphi} \) then the term structure is upward-sloping, as expected returns converge from \( x_t \) to their long-term value. The opposite is true when \( x_t > \frac{\mu}{1 - \varphi} \). In both cases the term structure increases or decreases with a decreasing rate as \( |\varphi| < 1 \). Using the estimates of the AR(1) model we can calculate this term structure for each day of our sample. We find that for 68\% of the days examined the term structure is upward-sloping. For the rest of the days, where the term structure is downward-sloping, they correspond to the two financial turmoil periods, i.e., the burst of the Dot-Com bubble in 2002 and the sub-prime mortgage crisis (2007 to 2009). During these periods the 1-month ahead expected return

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\(^{16}\)This empirical result is also consistent with several studies which also assumed that \( x_t \sim AR(1) \) process (see Cochrane (2008), *inter alia*).
was larger than its long-term value (see also Figure 3), indicating that, during financial turmoil periods, investors priced a significantly higher market risk in the short- than in the long-run. For illustration, Figure 4 shows the term structure of expected returns for an investment horizon up to 3 years on May 2005 (upward-sloping) and on November 2007 (downward-sloping).

5.2 Return decomposition

The return decomposition framework used in this paper is similar to that of Campbell (1991) with one important difference. Our decomposition is about unexpected returns without dividends instead of unexpected total returns. This is due to the fact that our model provides estimates of expected returns without dividends as the terminal payoff of European options depend on the index level net of the accrued realized dividends of it. This approach has also an advantage against the decomposition of unexpected total returns. It provides a closed-form solution, which does not relies on the log-linear approximation of Campbell and Shiller (1988).

The analysis is based on the following decomposition of unexpected returns:

\[
e_{t+i} = \frac{1}{(i/12)} \left( r_{t,t+i} - E_t^P [r_{t,t+i}] \right)
\]

\[
= \frac{1}{(i/12)} \sum_{j=0}^{\infty} \left( E_t^P \left[ \Delta d_{t+j,i,t+(j+1)i} \right] - E_t^P \left[ \Delta d_{t+j,i,t+(j+1)i} \right] \right) -
\]

\[
- \frac{1}{(i/12)} \sum_{j=1}^{\infty} \left( E_t^P \left[ r_{t+j,i,t+(j+1)i} \right] - E_t^P \left[ r_{t+j,i,t+(j+1)i} \right] \right)
\]

\[
= e_{CF,t+i} + e_{DR,t+i}
\]

where \( \Delta d_{t,t+i} = \ln \left( D_{t+i}/D_t \right) \) denotes the dividend growth rate between dates \( t \) and \( t + i \). This formula can be easily demonstrated by iterating forward the identity \( \ln S_t - \ln D_t = -r_{t,t+i} + \Delta d_{t,t+i} + \ln S_{t+i} - \ln D_{t+i} \), setting that \( \lim_{j \to \infty} (\ln S_{t+j} - \ln D_{t+j}) = 0 \), i.e., the price-dividend ratio does not explode, and finally take conditional expectations under the physical measure \( P \). Formula (22) indicates that unexpected returns \( e_{t+i} \) must be associated with changes in expectations of future cash flows or expected returns (also named as discount rates in the relevant literature). An increase in expected future cash flows is associated with a capital gain today, while an increase in future discount rates is associated with a capital
loss today. The two components, \( e_{CF,t+i} \) and \( e_{DR,t+i} \) define the cash flows (CF) news and the discount rates (DR) news, respectively.

In this paper we model \( e_{t,t+i} \) and \( e_{DR,t+i} \) for different horizons \( i \) using the term structure of expected returns implied by the AR(1) model presented in the previous section. Unexpected returns can be easily calculated as the difference between ex-post returns \( r_{t,t+i} \) and expected returns given by formula (20). DR news formula are given in the next proposition. CF news are then retrieved, following Campbell and Vuolteenaho (2004), as the difference between unexpected returns and DR news using formula (22).

**Proposition 3** If \( x_t \sim AR(1) \) model (18) then,

\[
e_{DR,t+i} = -\frac{1}{(i/12)} \left( \frac{\varepsilon_{t+i}}{1 - \varphi} + \frac{1}{1 - \varphi} \sum_{k=1}^{i-1} \varphi^{i-k} \varepsilon_{t+k} \right).
\]

To appraise the implications of formula (23) we examine it with respect to different investment horizons. For \( i = 1 \), formula (23) gives the DR news component for 1-month annualized unexpected returns. Then,

\[
e_{DR,t+1} = -12 \frac{1}{1 - \varphi} \varepsilon_{t+1}.
\]

Using the estimates of \( \varphi \) reported in (19) we have that \( 12/(1 - \hat{\varphi}) = 52.17 \). Thus, due to the persistence of \( x_t \), a positive shock in next month expected return measured by \( \varepsilon_{t+1} \), will have a significant negative impact on the 1-month return \( r_{t,t+1} \). The model predicts that if annualized expected returns unexpectedly rises at \( t + 1 \) by 1%, then the annualized return \( 12r_{t,t+1} \) will unexpectedly decrease by 52%. For \( i = 2 \), formula (23) gives the DR news component for 2-month annualized unexpected returns. This is equal to,

\[
e_{DR,t+2} = -\frac{1}{(2/12)} \left( \frac{\varphi}{1 - \varphi} \varepsilon_{t+1} + \frac{1}{1 - \varphi} \varepsilon_{t+2} \right).
\]

Using the estimates of \( \varphi \), we obtain an estimate of \( e_{DR,t+2} \) given as \(-20.08\varepsilon_{t+1} - 26.08\varepsilon_{t+2}\). Compared with the DR news of the 1-month return \( e_{DR,t+1} \), we observe that the coefficients have now decreased in magnitude. Thus, a positive 1% shock in monthly expected returns at time \( t+1 \) and \( t+2 \) will make the 2-month annualized return \( 1/(2/12) r_{t,t+2} \) to unexpectedly decrease by 46%, which is smaller than the 52% decrease observed for \( 12r_{t,t+1} \).
The previous discussion is also related to the fact that the variance of \( e_{DR,t+i} \) converges to zero as \( i \) tends to infinity. Formally, one can easily deduce from formula (23) that:

\[
Var(e_{DR,t+i}) = f(i; \varphi)\sigma^2,
\]

where \( \sigma^2 = Var(\varepsilon_{t+1}) \) and

\[
f(i; \varphi) = \frac{1}{(i/12)^2} \left( \frac{1}{(1 - \varphi)^2} \left( 1 - \varphi^{2i} \right) \right).
\]

Then, it can be shown that function \( f(i; \varphi) \) is decreasing with respect to \( i \) and \( \lim_{i \to \infty} f(i; \varphi) = 0 \). Thus, as investment horizon \( i \) increases the effect of DR news on the variation of annualized returns decreases, whereas the effect of CF news should respectively increases. In the limiting case of \( i \to \infty \), DR news has no effect on the variation of long-term annualized returns. The latter, is fully due to CF news. It is important to stress here that this result does not depend on the time-series model, i.e., the \( AR(1) \), that we choose for \( x_t \). It is due to the assumption that \( x_t \) follows a stationary process, which implies that the long-term annualized expected return is time-invariant (see equation (21)). Thus, any shock in long-term returns can only be attributed to shocks on expected dividend growth rates.

Formula (22) also implies that:

\[
1 = \frac{Cov(e_{t+i}, e_{CF,t+i})}{Var(e_{t+i})} + \frac{Cov(e_{t+i}, e_{DR,t+i})}{Var(e_{t+i})} = b_{CF} + b_{DR}
\]

This identity allows us to decompose unexpected returns variation into two parts, denoted as \( b_{CF} \) and \( b_{DR} \), respectively. The relative magnitude of these two coefficients indicates which of these two shocks attributes more to the observed variation of unexpected returns.

Table 4 reports the empirical results of return decomposition. Panel A1 reports the relevant statistics, defined previously, for the full sample period, December 2001 to October 2010, while Panel A2 reports the same statistics excluding the last part of our sample (i.e., from August 2007 to October 2010), which covers the recent financial crisis. Several conclusions can be drawn from the results reported in the table. First, Panel A clearly indicates that, for our sample period, shocks in realized returns can be mainly attributed to CF news. For 1-month horizon, 68% of the variation in unexpected returns is due to CF news, while 32%
to DR news. As expected, the relative magnitude of CF news increases with investment horizon. Thus, 3-year unexpected returns variation is almost entirely attributed to changes in expected dividend growth rates. Second, these results remain robust even when the recent financial crisis is excluded from the sample. Again, for the 1-month investment horizon 70% of the variation in unexpected returns is due to CF news, while 30% to DR news. To further examine changes in CF and DR news during our sample period, Figure 5 presents smoothed exponentially weighted moving average of 1-month DR and CF news (see also Campbell and Vuolteenaho (2004)). The graphs of the figure indicate that in the recent recession period stock market decline can be attributed both to negative CF (decreasing cash flows) and DR (increasing discount rates) news.\(^\text{17}\) Campbell and Vuolteenaho (2004) defined that as a "mixed recession". We also observe that negative DR news, appeared at the end of 2008, precede negative CF news, which are observed at the beginning of 2009.

Chen and Zhao (2009) argued that the approach of directly estimating DR news and back out the CF news using formula (22), as suggested by Campbell and Vuolteenaho (2004), has a serious limitation. CF news could inherit the misspecification error of DR news. To overcome this limitation they suggested to model both CF and DR news. Such a procedure effectively separates the CF news into modeled CF news and residual news. The nature of the residual news is unknown but we can assess the relative magnitude of modeled CF and DR news in explaining unexpected returns variation equally. Under this approach formula (22) can be revised as follows:

\[
e_{t+i} = e_{CF,t+i} + e_{DR,t+i} + e_{res,t+i},
\]

where \(e_{t+i}\) and \(e_{DR,t+i}\) are the same as before, \(e_{CF,t+i}\) is now explicitly estimated and \(e_{res,t+i}\) represents the component of the unexpected return that is not captured by the modeled CF and DR news. To estimate \(e_{CF,t+i}\) we follow the relevant literature (see Chen and Zhao

\(\text{Note here that we have defined DR news as}
\]

\[
e_{DR,t+i} = -\frac{1}{(t/12)} \sum_{j=1}^{\infty} (E_t^P [r_{t+j,i,t+(j+1)i}] - E_t^P [r_{t+j,i,t+(j+1)i}])
\]

So, when shocks in discount rates increase, \(e_{DR,t+i}\) decreases.
(2009), *inter alia*) and we adopt the VAR system:

\[ z_{t+1} = \mu_z + \Gamma z_t + w_{t+1}, \tag{25} \]

where \( z_{t+1} \) contains the 1-month dividend growth rate, the 1-month stock log-return and the logarithm of the dividend-price ratio, denoted as \( dp_{t+1} \), thus \( z_{t+1} = (\Delta d_{t,t+1}, r_{t,t+1}, dp_{t+1})' \).

Then, the 1-month CF news component is given as:

\[ e_{CF,t+1} = 12e1' (I - \Gamma)^{-1} w_{t+1}, \tag{26} \]

where \( e1 = (1, 0, 0)' \). Iterating forward this formula we can deduce CF news for an investment horizon equal to \( i \) months as follows:

\[ e_{CF,t+i} = \frac{e1'}{(i/12)} \left( \sum_{j=1}^{i} \left( \sum_{k=j}^{i} \Gamma^{k-j} + \Gamma^{i+1-j} (I - \Gamma)^i \sum_{k=1}^{i} \Gamma^{k-1} \right) w_{t+i} \right). \tag{27} \]

The proof of this formula can be provided by the authors upon request. Using monthly S&P 500 index data on \( z_{t+1} \) from December 2001 to October 2010 obtained from Robert Shiller’s website (http://www.econ.yale.edu/~shiller/data.htm) we estimate (25). These estimates are reported in the Table 5. Then, using formula (27) we estimate CF news. Finally, we construct \( e_{res,t+i} \) by substituting the estimates of \( e_{t+i}, e_{CF,t+i} \) and \( e_{DR,t+i} \) into formula (24). The results of this return decomposition approach are reported in Panel B of Table 4.

These results indicate that CF news are still the dominant factor is explaining the variation of unexpected returns for our sample period. For example, for 1-month horizon 50.5% of the variation in \( e_{t+i} \) is due to CF news. As expected the relative importance of CF news increases with respect to the investment horizon. Figure 5 also presents smoothed exponentially weighted moving average of 1-month modeled CF news. We observe that this series moves closely to CF news implied by DR news, albeit with a lower variability. These results also indicate that 17.5% of 1-month unexpected returns variation cannot be explained by the estimated CF and DR news. This could be due to misspecification errors encountered in the estimation of CF and DR news, or on dividend smoothing observed in the post-war data which weakens dividend growth rates predictability (see Chen, Da and Priestley (2012)). Nevertheless, the previous conclusion is still valid. Changes in the
expectations of dividend growth rates is the dominant component of unexpected returns variation.

To understand why modeled CF news support this conclusion we need to look at the estimates of VAR system (25) reported in Panel A of Table 5. The important result in this table is the negative, equal to $-0.008$, and statistically significant coefficient of $dp_t$ on $\Delta d_{t,t+1}$. This estimate indicates that dividend-price ratio predicts 1-month ahead dividend growth rates with the correct sign (when $dp_t$ increases, which means that prices decrease, then next month dividends tend also to decrease) in our sample. Thus, the variation of $e_{CF,t+1}$ is due to the fact that $dp_t$, which is a highly persistent variable (the estimated autoregressive coefficient is 0.97), predicts 1-month ahead dividend growth rates.

For comparison reasons we also report the results of return decomposition based on Campbell and Vuolteenaho (2004) approach. More precisely, using VAR system (25) we estimate $e_{t+1}$ and $e_{DR,t+1}$ and we backed out $e_{CF,t+1}$ using formula (22). We perform this return decomposition using data from the whole sample period (12/2001-10/2010) and the subsample period which excludes the recent financial crisis (12/2001-7/2007). These results are reported in Table 5. For the whole sample period we observe that these results are in accordance with those reported in Table 4. For 1-month horizon, CF news are the dominant factor of unexpected return variation. Again, this is due to the fact that, for this sample period, $dp_t$ predicts dividend growth rates and not returns. When however we perform this analysis for the subsample period which does not include the recent financial crisis we obtain the opposite results as 89% of unexpected return variation is due to DR news. This conclusion can again be explained by the estimates of the VAR system. For this subsample period $dp_t$ predicts returns and not dividend growth rates.\textsuperscript{18} We acknowledge that other state variables, apart from the dividend-price ratio, are also suggested in the literature to predict returns and dividend growth rates. However, as already shown in the literature (see Chen and Zhao (2009)), this will not improve the reliability of this approach.

Overall, we conclude that our approach indicates that CF news is the dominant factor of unexpected return variation for investment horizons larger or equal than 1-month. This result is robust to whether or not the recent financial crisis is included in the sample period or CF news are directly modeled or backed out from DR news. On the other hand, a

\textsuperscript{18}The sensitivity of predictive regressions estimates to the sample period is a well-documented disadvantage of this approach, already reported in the literature (see Chen, Da and Zhao (2013), \textit{inter alia}).
predictive regression approach provides confounding results for the relative importance of DR and CF news in driving the variation of stock returns. It worth noticing however that, for the whole sample period, the two approaches lead to similar conclusions.

6 Conclusion

This paper develops a new methodology to estimate the market expected return and the associated ex-ante market risk premium combining information from the stock and option data. Assuming a monotonic pricing kernel, motivated by the power/logarithmic utility function, we first derive a theoretical model that relates physical cumulants of the market log-return of any order to risk-neutral ones through the projected relative risk aversion coefficient. This model implies that the ex-ante market risk premium depends on higher-order physical or risk-neutral cumulants and the projected relative risk aversion coefficient. The model implies that higher-order physical and risk-neutral cumulants are also related through this coefficient. Essentially, this method estimates the market expected return (i.e., the mean of the physical distribution) at every time period so that the dispersion between the physical and risk-neutral distribution can be explained by a monotonic pricing kernel. It can thought as a generalization of Duan and Zhang’s (2014) approach which exploits a larger set of properties of the imposed theoretical model.

In the empirical part of the paper using forward-looking estimates of higher-order physical cumulants retrieved from the S&P 500 daily index returns and risk-neutral ones implied from European option prices written on this index and a system GMM estimation procedure, we estimate the projected relative risk aversion coefficient in a monthly frequency from 2001 to 2010. Then, combining the estimates of this coefficient along with the estimates of higher-order physical or risk-neutral cumulants we retrieve the ex-ante market risk premium at every time period. The estimated market risk premium is always positive, time-varying and counter-cyclical, showing high variability, during the sample period, from around 1% to around 100%. As expected it increases during the two financial turmoil periods, i.e., the burst of the Dot-Com bubble in 2002 and the sub-prime mortgage crisis (2007 to 2009).

The estimation of a time series of market expected returns enables us to examine if stock prices move due to shocks in expected returns or expected cash flows, and how much of each. In contrast to the prevailing belief, our results indicate that shocks in cash flows is the dominant factor of unexpected return variation for investment horizons larger or equal
than 1-month for our sample period. This finding is robust to whether or not the recent financial crisis is included in the sample period or shocks in cash flows are directly modeled or backed out from the observed shocks in expected returns. These results complement an increasing number of recent studies that demonstrate the importance of shocks in cash flows in asset pricing.

A Appendix

In this appendix, we provide proofs of the main theoretical results of the paper

A.1 Proof of Proposition 1

To prove formula (7) set $u - \gamma = v$ in (6). Then we have:

$$m_t^Q (v + \gamma) = \ln \beta + r_f \tau + m_t^P (v).$$

The $n^{th}$-order derivative of the last relationship with respect to $v$ evaluated at $v = 0$ gives

$$\left[ \frac{d^n m_t^Q (v + \gamma)}{dv^n} \right]_{v=0} = \left[ \frac{d^n m_t^P (v)}{dv^n} \right]_{v=0}, \quad \forall \ n \in \mathbb{N}. \quad (28)$$

By definition the $n^{th}$-order cumulant of $r_{t,T}$ under the physical measure is equal to $k_{r,n}^P = \left[ \frac{d^n m_t^P (v)}{dv^n} \right]_{v=0}$. Also writing $m_t^Q (v + \gamma)$ in a power series expansion as

$$m_t^Q (v + \gamma) = \sum_{m=1}^{\infty} k_{r,m}^Q (v + \gamma)^m / m! \quad (29)$$

and calculating the $n^{th}$-order derivative yields

$$\left[ \frac{d^n m_t^Q (v + \gamma)}{dv^n} \right]_{v=0} = \sum_{m=0}^{\infty} k_{r,n+m}^Q \gamma^m / m!, \quad (30)$$

Substituting formulas (29) and (30) into (28) yields

$$k_{r,n}^P = \sum_{m=0}^{\infty} k_{r,n+m}^Q \gamma^m / m!, \quad \forall \ n \in \mathbb{N},$$

which is exactly formula (7).
To prove formula (8) take the $n^{th}$-order derivative of (6) with respect to $u$ evaluated at $u = 0$ which gives:

$$
\left[ \frac{d^n m_t^Q(u)}{du^n} \right]_{u=0} = \left[ \frac{d^n m_t^P(u - \gamma)}{du^n} \right]_{u=0}, \forall \ n \in \mathbb{N}. \quad (31)
$$

By definition the $n^{th}$-order cumulant of $r_t,T$ under the risk-neutral measure is equal to $k_{r,n}^Q = \left[ \frac{d^n m_t^Q(u)}{du^n} \right]_{u=0}$. Also writing $m_t^P(u - \gamma)$ in a power series expansion as

$$
m_t^P(u - \gamma) = \sum_{m=1}^{\infty} k_{r,m}^P \frac{(u - \gamma)^m}{m!} \quad (32)
$$

and calculating the $n^{th}$-order derivative yields

$$
\left[ \frac{d^n m_t^P(u - \gamma)}{du^n} \right]_{u=0} = \sum_{m=0}^{\infty} k_{r,n+m}^P \frac{(-\gamma)^m}{m!}. \quad (33)
$$

Substituting formulas (32) and (33) into (31) yields

$$
k_{r,n}^Q = \sum_{m=0}^{\infty} k_{r,n+m}^P \frac{(-\gamma)^m}{m!}, \quad \forall \ n \in \mathbb{N},
$$

which is exactly formula (8).

### A.2 Proof of Proposition 2

By the law of iterated expectations we have that:

$$
E_t^P [r_{t,t+j}] = E_t^P \left[ \sum_{i=0}^{j-1} r_{t+i,t+i+1} \right] = E_t^P \left[ \sum_{i=0}^{j-1} E_{t+i}^P [r_{t+i,t+i+1}] \right] =
$$

$$
= E_t^P \left[ \sum_{i=0}^{j-1} x_{t+i} \right] = x_t + \sum_{i=1}^{j-1} E_t^P [x_{t+i}], \quad (34)
$$

The AR(1) model (18) implies that:

$$
E_t^P [x_{t+i}] = \mu \frac{1}{1 - \varphi} + \varphi^i x_t. \quad (35)
$$
Substituting formula (35) into (34) we obtain:

\[
E_t^P [r_{t,i+j}] = x_t + \sum_{i=1}^{j-1} \left( \mu \frac{1-\varphi^i}{1-\varphi} + \varphi^i x_t \right) = \frac{\mu}{1-\varphi} \left( j - \frac{1-\varphi^j}{1-\varphi} \right) + \frac{1-\varphi^j}{1-\varphi} x_t.
\]

Thus,

\[
\frac{1}{(j/12)} E_t^P [r_{t,i+j}] = \frac{12\mu}{1-\varphi} + \frac{1}{(j/12)} \frac{1-\varphi^j}{1-\varphi} \left( x_t - \frac{\mu}{1-\varphi} \right),
\]

which is exactly formula (20). By taking the limit of the annualized expected return as \( j \) approaches infinity we can easily deduce formula (21).

A.3 Proof of Proposition 3

First note that we have shown that the AR(1) model (18) implies that:

\[
E_t^P [x_{t+i}] = \mu \frac{1-\varphi^i}{1-\varphi} + \varphi^i x_t.
\]

(see formula (35)). We decompose \( e_{DR,t+i} \) as:

\[
e_{DR,t+i} = -\frac{1}{(i/12)} (E_{t+i}^P [r_{t+i,t+2i}] - E_t^P [r_{t+i,t+2i}]) - \frac{1}{(i/12)} \sum_{j=2}^{\infty} (E_{t+i}^P [r_{t+ji,t+(j+1)i}] - E_t^P [r_{t+ji,t+(j+1)i}]).
\]

For the first term of (37) we have that:

\[
E_{t+i}^P [r_{t+i,t+2i}] = E_{t+i}^P \left[ \sum_{k=1}^{i} r_{t+i+k-1,t+i+k} \right] = E_{t+i}^P \left[ \sum_{k=1}^{i} E_{t+i+k-1}^P [r_{t+i+k-1,t+i+k}] \right]
\]

\[
= E_{t+i}^P \left[ \sum_{k=1}^{i} x_{t+i+k-1} \right] = x_{t+i} + \sum_{k=2}^{i} E_{t+i}^P [x_{t+i+k-1}].
\]
For \( l = k - 1 \) and \( t \equiv t + i \) formula (36) gives:

\[
E_{t+i}^P [x_{t+i+k-1}] = \mu \frac{1 - \varphi^{k-1}}{1 - \varphi} + \varphi^{k-1}x_{t+i}.
\]

Thus, the previous formula can be written as:

\[
E_{t+i}^P [r_{t+i,t+2i}] = x_{t+i} + \sum_{k=2}^{i} \mu \frac{1 - \varphi^{k-1}}{1 - \varphi} + \varphi^{k-1}x_{t+i} = \\
= \frac{\mu}{1 - \varphi} \left( i - \frac{1 - \varphi^i}{1 - \varphi} \right) + \frac{1 - \varphi^i}{1 - \varphi}x_{t+i},
\]

which also implies that:

\[
E_t^P [r_{t+i,t+2i}] = E_t^P [E_{t+i}^P [r_{t+i,t+2i}]] = \\
= \frac{\mu}{1 - \varphi} \left( i - \varphi^i \frac{1 - \varphi^i}{1 - \varphi} \right) + \varphi^i \frac{1 - \varphi^i}{1 - \varphi}x_t.
\]

Thus,

\[
E_{t+i}^P [r_{t+i,t+2i}] - E_t^P [r_{t+i,t+2i}] = -\mu \left( \frac{1 - \varphi^i}{1 - \varphi} \right)^2 + \frac{1 - \varphi^i}{1 - \varphi} (x_{t+i} - \varphi^i x_t).
\]

Given that,

\[
x_{t+i} = \mu \frac{1 - \varphi^i}{1 - \varphi} + \varphi^i x_t + \sum_{k=1}^{i-1} \varphi^{i-k} \varepsilon_{t+k} + \varepsilon_{t+i},
\]

we can write the last formula as:

\[
E_{t+i}^P [r_{t+i,t+2i}] - E_t^P [r_{t+i,t+2i}] = \frac{1 - \varphi^i}{1 - \varphi} \left( \sum_{k=1}^{i-1} \varphi^{i-k} \varepsilon_{t+k} + \varepsilon_{t+i} \right).
\]
For the second term in the right-hand-side of formula (37) we have that:

\[
E_{t+i}^P \left[ r_{t+j_i, t+(j+1)i} \right] = E_{t+i}^P \left[ \sum_{k=1}^{i} r_{t+j_i+k-1, t+j_i+k} \right] =
\]
\[
= E_{t+i}^P \left[ \sum_{k=1}^{i} E_{t+j_i+k-1}^P [r_{t+j_i+k-1, t+j_i+k}] \right] =
\]
\[
= E_{t+i}^P \left[ \sum_{k=1}^{i} x_{t+j_i+k-1} \right] =
\]
\[
\sum_{k=1}^{i} E_{t+i}^P [x_{t+j_i+k-1}] . \tag{40}
\]

Thus for \( l = (j-1)i + k - 1 \) and \( t \equiv t + i \) formula (36) gives:

\[
E_{t+i}^P [x_{t+j_i+k-1}] = \mu \frac{1 - \varphi^{(j-1)i+k-1}}{1 - \varphi} + \varphi^{(j-1)i+k-1} x_{t+i}.
\]

Substituting the last formula into (40) and making algebraic calculations yields:

\[
E_{t+i}^P \left[ r_{t+j_i, t+(j+1)i} \right] = \mu \frac{1 - \varphi^{i(j-1)} \varphi^i}{1 - \varphi} + \varphi^{i(j-1)} \frac{1 - \varphi^i}{1 - \varphi} x_{t+i}.
\]

Then,

\[
E_t^P \left[ r_{t+j_i, t+(j+1)i} \right] = E_t^P \left[ E_{t+i}^P \left[ r_{t+j_i, t+(j+1)i} \right] \right] =
\]
\[
= \frac{\mu}{1 - \varphi} \left( i - \varphi^i \frac{1 - \varphi^i}{1 - \varphi} \right) + \varphi^i \frac{1 - \varphi^i}{1 - \varphi} x_t.
\]

Thus, using formula (38) we can write that:

\[
E_{t+i}^P \left[ r_{t+j_i, t+(j+1)i} \right] - E_t^P \left[ r_{t+j_i, t+(j+1)i} \right] =
\]
\[
\varphi^{i(j-1)} \frac{1 - \varphi^i}{1 - \varphi} \left( \varepsilon_{t+i} + \sum_{k=1}^{i-1} \varphi^{i-k} \varepsilon_{t+k} \right).
\]

Thus, the second term in the right-hand-side of (37) is equal to:

\[
\sum_{j=2}^{\infty} \left( E_{t+i}^P \left[ r_{t+j_i, t+(j+1)i} \right] - E_t^P \left[ r_{t+j_i, t+(j+1)i} \right] \right) =
\]
\[
\sum_{j=2}^{\infty} \varphi^{i(j-1)} \frac{1 - \varphi^i}{1 - \varphi} \left( \varepsilon_{t+i} + \sum_{k=1}^{i-1} \varphi^{i-k} \varepsilon_{t+k} \right). \tag{41}
\]
Formulas (39) and (41) implies that:

\[
\epsilon_{DR,t+i} = -\frac{1}{(i/12)} \left[ \left( \frac{1-\varphi^i}{1-\varphi} \left( \epsilon_{t+i} + \sum_{k=1}^{i-1} \varphi^{-k} \epsilon_{t+k} \right) \right) + \right.
\]
\[
+ \sum_{j=2}^{\infty} \varphi^{(j-1)} \frac{1-\varphi^i}{1-\varphi} \left( \epsilon_{t+i} + \sum_{k=1}^{i-1} \varphi^{-k} \epsilon_{t+k} \right) \right]
\]
\[
= -\frac{1}{(i/12)} \left[ \left( \frac{1-\varphi^i}{1-\varphi} + \sum_{j=2}^{\infty} \varphi^{(j-1)} \frac{1-\varphi^i}{1-\varphi} \right) \epsilon_{t+i} + \right.
\]
\[
+ \frac{1-\varphi^i}{1-\varphi} \sum_{j=1}^{\infty} \varphi^{(j-1)} \sum_{k=1}^{i-1} \varphi^{-k} \epsilon_{t+k} \right]
\]
\[
= -\frac{1}{(i/12)} \left[ \left( \frac{1-\varphi^i}{1-\varphi} + \sum_{j=1}^{\infty} \varphi^{(j-1)} \right) \epsilon_{t+i} + \frac{1}{1-\varphi} \sum_{k=1}^{i-1} \varphi^{-k} \epsilon_{t+k} \right]
\]
\[
= -\frac{1}{(i/12)} \left( \epsilon_{t+i} + \frac{1}{1-\varphi} \sum_{k=1}^{i-1} \varphi^{-k} \epsilon_{t+k} \right),
\]

which proves formula (23).
References


Table 1: Descriptive statistics

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<th>Risk-neutral</th>
<th>Physical</th>
<th>Correlation (%)</th>
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<td>Variance</td>
<td>0.004638</td>
<td>0.003147</td>
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<td></td>
<td>(0.005310)</td>
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<td>3rd-order cumulant</td>
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<td></td>
<td>(0.000398)</td>
<td>(0.000224)</td>
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This table reports average values of second, third and fourth-order cumulants of the physical and risk-neutral distribution. Standard deviations are in parentheses. The last column of the table presents the correlation coefficient between the respective risk-neutral and physical cumulants. Risk-neutral cumulants are estimated directly from S&P 500 index option prices with approximately 1 month time-to-maturity using equation (15). Physical cumulants are estimated by the FHS method employed to daily index returns. The estimation period span from January 1996 to October 2010.
Table 2: Projected risk aversion coefficient estimates

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<td>0.43</td>
<td>0.53</td>
<td>0.43</td>
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<td>0.36</td>
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<td>p-value</td>
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<td>0.51</td>
<td>0.51</td>
<td>0.50</td>
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<td>0.36</td>
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Panel B: Model (16)

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<td>Avg.</td>
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<td>1.93</td>
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<td>2.15</td>
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<td>9.83</td>
<td>5.78</td>
<td>5.57</td>
<td>7.71</td>
<td>9.10</td>
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<td>5.57</td>
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<td>SE</td>
<td>Avg.</td>
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<td>0.49</td>
<td>0.70</td>
<td>0.73</td>
<td>0.70</td>
<td>0.26</td>
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<td>0.32</td>
<td>0.27</td>
<td>0.18</td>
<td>0.11</td>
</tr>
<tr>
<td>p-value</td>
<td>Avg.</td>
<td>0.22</td>
<td>0.36</td>
<td>0.54</td>
<td>0.67</td>
<td>0.38</td>
<td>0.24</td>
<td>0.37</td>
<td>0.35</td>
<td>0.51</td>
<td>0.23</td>
<td>0.32</td>
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<tr>
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<td>0.02</td>
<td>0.01</td>
<td>0.12</td>
<td>0.03</td>
<td>0.04</td>
<td>0.03</td>
<td>0.03</td>
<td>0.11</td>
<td>0.10</td>
<td>0.10</td>
</tr>
</tbody>
</table>

This table reports descriptive statistics of the GMM estimation results for both specifications, (16) and (17) and for different values of $N$ and $M$. More precisely, it reports the average, standard deviation, minimum and maximum values of the estimates of $\gamma$. It also reports the average standard error (denoted as SE) and the average and minimum values of $p$-value of Hansen’s overidentified restriction test. For each observation date we use a 6-year moving window of data to estimate $\gamma$. The instruments used with orthogonality conditions (17) are the constant and one, two and three periods lagged values of $k_{r,2}^Q$, $k_{r,3}^Q$, $k_{r,4}^Q$ and $k_{r,5}^Q$ for the first, second, third and fourth equation of the system, respectively. The instruments used with orthogonality conditions (16) are the constant and one, two and three periods lagged values of $k_{r,2}^P$, $k_{r,3}^P$, $k_{r,4}^P$ and $k_{r,5}^P$ for the first, second, third and fourth equation of the system, respectively. Standard errors are corrected for heteroskedasticity and serial correlation. The estimation period span from December 2001 to October 2010.
This table reports sample descriptive statistics of the annualized ex-ante market risk premium estimates (in percentage points) of the S&P 500 index for the different model specifications used for the estimation of the projected relative risk aversion coefficient. It reports the average, standard deviation, minimum and maximum values of these estimates and their first-order autocorrelation, denoted as $R(-1)$. Panel A reports estimates of the ex-ante market risk premium based on estimates of $\gamma$ retrieved from specification (17) and the third-order approximation of formula (9), while Panel B reports estimates of it based on a third-order approximation of formula (10). In this latter case the GMM estimates of $\gamma$ are based on the orthogonality conditions (16). The estimation period span from December 2001 to October 2010.
Table 4: Return decomposition of the S&P 500 index

<table>
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<tr>
<th>Horizon (in months)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>24</th>
<th>36</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: DR news directly modeled</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$SD(e_{t+i})$</td>
<td>62.5</td>
<td>49.5</td>
<td>41.9</td>
<td>32.5</td>
<td>26.6</td>
<td>22.5</td>
<td>13.1</td>
<td>9.12</td>
</tr>
<tr>
<td>$SD(e_{CF,t+i})$</td>
<td>49.1</td>
<td>39.1</td>
<td>33.5</td>
<td>27.8</td>
<td>23.8</td>
<td>20.7</td>
<td>12.8</td>
<td>9.13</td>
</tr>
<tr>
<td>$SD(e_{DR,t+i})$</td>
<td>31.8</td>
<td>18.6</td>
<td>14.2</td>
<td>8.15</td>
<td>5.25</td>
<td>3.97</td>
<td>2.09</td>
<td>1.47</td>
</tr>
<tr>
<td>$Cor(e_{CF,t+i}, e_{DR,t+i})$</td>
<td>15.2</td>
<td>39.8</td>
<td>44.9</td>
<td>49.1</td>
<td>46.6</td>
<td>38.6</td>
<td>4.45</td>
<td>−8.52</td>
</tr>
<tr>
<td>$b_{CF}$</td>
<td>68</td>
<td>74.1</td>
<td>76.3</td>
<td>83.3</td>
<td>87.9</td>
<td>90.7</td>
<td>96.7</td>
<td>98.8</td>
</tr>
<tr>
<td>$b_{DR}$</td>
<td>32</td>
<td>25.9</td>
<td>23.7</td>
<td>16.7</td>
<td>12.1</td>
<td>9.34</td>
<td>3.27</td>
<td>1.24</td>
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<tr>
<td>$SD(e_{t+i})$</td>
<td>42.7</td>
<td>32.4</td>
<td>28.3</td>
<td>21</td>
<td>15.6</td>
<td>12</td>
<td>4.26</td>
<td>3.61</td>
</tr>
<tr>
<td>$SD(e_{CF,t+i})$</td>
<td>36</td>
<td>25</td>
<td>21.4</td>
<td>16.7</td>
<td>13.8</td>
<td>11.3</td>
<td>4.3</td>
<td>3.69</td>
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<tr>
<td>$SD(e_{DR,t+i})$</td>
<td>23.4</td>
<td>13.9</td>
<td>10.4</td>
<td>6.26</td>
<td>2.76</td>
<td>1.43</td>
<td>0.46</td>
<td>0.21</td>
</tr>
<tr>
<td>$Cor(e_{CF,t+i}, e_{DR,t+i})$</td>
<td>−1.21</td>
<td>33.1</td>
<td>52.3</td>
<td>59.8</td>
<td>61.3</td>
<td>46.9</td>
<td>−13.8</td>
<td>−42.3</td>
</tr>
<tr>
<td>$b_{CF}$</td>
<td>70.5</td>
<td>70.5</td>
<td>71.9</td>
<td>77</td>
<td>87.3</td>
<td>93.3</td>
<td>100.3</td>
<td>102.2</td>
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<tr>
<td>$b_{DR}$</td>
<td>29.5</td>
<td>29.5</td>
<td>28.1</td>
<td>23</td>
<td>12.7</td>
<td>6.66</td>
<td>−0.34</td>
<td>−2.19</td>
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<tr>
<td><strong>Panel B: DR and CF news directly modeled (12/2001-10/2010)</strong></td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>$SD(e_{CF,t+i})$</td>
<td>48.1</td>
<td>34.4</td>
<td>27.4</td>
<td>21.3</td>
<td>17.8</td>
<td>15.7</td>
<td>11.2</td>
<td>8.35</td>
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<tr>
<td>$SD(e_{res,t+i})$</td>
<td>41.5</td>
<td>25.7</td>
<td>19.5</td>
<td>11.8</td>
<td>9.01</td>
<td>7.30</td>
<td>3.16</td>
<td>2.13</td>
</tr>
<tr>
<td>$Cor(e_{CF,t+i}, e_{DR,t+i})$</td>
<td>30.5</td>
<td>50.3</td>
<td>60.1</td>
<td>60.6</td>
<td>52.8</td>
<td>47</td>
<td>12.7</td>
<td>−1.85</td>
</tr>
<tr>
<td>$b_{CF}$</td>
<td>50.5</td>
<td>54.9</td>
<td>56</td>
<td>61.3</td>
<td>63.5</td>
<td>67</td>
<td>83</td>
<td>89</td>
</tr>
<tr>
<td>$b_{res}$</td>
<td>17.5</td>
<td>19.2</td>
<td>20.3</td>
<td>22</td>
<td>24.4</td>
<td>23.7</td>
<td>13.7</td>
<td>9.8</td>
</tr>
</tbody>
</table>

The table reports the standard deviation, denoted as $SD$, of unexpected returns $e_{t+i}$, CF news $e_{CF,t+i}$ and DR news $e_{DR,t+i}$ for the S&P 500 index for different investment horizons. It also reports the correlation coefficient between DR and CF news and the estimates of the coefficients $b_{CF}$ and $b_{DR}$, which measures the relative magnitude of CF and DR news, respectively, to unexpected returns variation. Panel A reports the results when DR news are directly modeled and CF news are backed out using formula (22) for the full sample period, December 2001 to October 2010 (see Panel A1), and for the subsample period, December 2001 to July 2007 (see Panel A2). Panel B reports the results when CF news are also directly modeled using VAR system (25). Data are sampled in a monthly frequency. All statistics are reported in percentage points.
Table 5: Return decomposition based on predictive regressions

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<td><strong>Panel A: VAR estimates</strong></td>
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<tr>
<td>$\mu_z$</td>
<td>$\Delta d_{t,t+1}$</td>
<td>$r_{t,t+1}$</td>
</tr>
<tr>
<td></td>
<td>-0.03</td>
<td>0.06</td>
</tr>
<tr>
<td></td>
<td>[-4.22]</td>
<td>[0.48]</td>
</tr>
<tr>
<td>$\Delta d_{t-1,t}$</td>
<td>0.83</td>
<td>-0.1</td>
</tr>
<tr>
<td></td>
<td>[20.85]</td>
<td>[-0.16]</td>
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<tr>
<td>$r_{t-1,t}$</td>
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<td>[2.25]</td>
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<td>$d_{p,t}$</td>
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<tr>
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<td>[-4.25]</td>
<td>[0.47]</td>
</tr>
<tr>
<td>$R^2$</td>
<td>91%</td>
<td>5.1%</td>
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<tr>
<td>$\bar{\sigma}$</td>
<td>0.003</td>
<td>0.046</td>
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<table>
<thead>
<tr>
<th><strong>Panel B: Return decomposition</strong></th>
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</thead>
<tbody>
<tr>
<td>$SD(e_{t+1})$</td>
<td>$SD(e_{CF,t+1})$</td>
<td>$SD(e_{DR,t+1})$</td>
</tr>
<tr>
<td>55.2</td>
<td>48.2</td>
<td>8.94</td>
</tr>
<tr>
<td>$Cor(e_{CF,t+1}, e_{DR,t+1})$</td>
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<td>$b_{DR}$</td>
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<td>75.3</td>
<td>86.7</td>
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<tr>
<td>$SD(e_{t+1})$</td>
<td>$SD(e_{CF,t+1})$</td>
<td>$SD(e_{DR,t+1})$</td>
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<tr>
<td>39.4</td>
<td>16</td>
<td>38.4</td>
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</tbody>
</table>

The table reports the results of return decomposition based on predictive regressions. Panel A reports OLS estimates of (25) with $t$-statistics in square brackets. The system includes a constant, the 1-month dividend growth rate ($\Delta d_{t,t+1}$), 1-month log-return ($r_{t,t+1}$) and log dividend-price ratio ($d_{p,t+1}$). Each column corresponds to a different dependent variable. The first four rows report coefficients on the four explanatory variables, the fifth row shows the $R^2$ coefficient, and the last row shows the estimates of the standard deviation of residuals. Sample periods are from December 2001 to October 2010 and from December 2001 to July 2007. The data are sampled monthly. Panel B reports the properties of 1-month DR and CF news implied by the estimates of VAR system (25) reported in Panel A. All statistics are reported in percentage points.
Figure 1: Time series monthly estimates of conditional variance, third and fourth-order cumulants of log-return distribution under physical and risk-neutral measures from January 1996 to October 2010.
Relative risk aversion coefficient estimates

Figure 2: Projected relative risk aversion coefficient estimates, December 2001 to October 2010.
Figure 3: Annualized ex-ante risk premium estimates of the S&P 500 index, December 2001 to October 2010.
Figure 4: Term structure of annualized expected returns on May 2005 and on November 2007. The horizontal line represents the long-term annualized expected return.
Figure 5: 1-month CF (left axis) and DR (right axis) news smoothed with an exponentially weighted moving average, December 2001 to October 2010.